Flavor Physics and Right-Handed Models

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Abstract

The Standard Model of particle physics only provides a parametrization of flavor which involves the values of the quark and lepton masses and unitary flavor mixing matrix i.e. CKM (Cabibbo-Kobayashi-Masakawa) matrix for quarks. The precise determination of elements of the CKM matrix is important for the study of the flavor sector of quarks. Here we concentrate on the matrix element $|V_{cb}|$. In particular we consider the effects on the value of $|V_{cb}|$ from possible right-handed admixtures along with the usually left-handed weak currents.

Left Right Symmetric Model provide a natural basis for right-handed current contributions and has been studied extensively in the literature but has never been discussed including flavor. In the first part of the present work an additional flavor symmetry is included in LRSM which allows a systematic study of flavor effects. The second part deals with the practical extraction of a possible right-handed contribution. Starting from the quark level transition $b \to c$ we use heavy quark symmetries to relate the helicities of the quarks to experimentally accessible quantities. To this end we study the decays $\bar{B} \to D(D^*)l\bar{\nu}$ which have been extensively explored close to non recoil point. By taking into account SCET (Soft Collinear Effective Theory) formalism it has been extended to a maximum recoil point i.e. $v \cdot v' \gg 1$. We derive a factorization formula , where the set of form factors is reduced to a single universal form factor $\xi(v \cdot v')$ up to hard-scattering corrections. Symmetry relations on form factors for exclusive $\bar{B} \to D(D^*)l\bar{\nu}$ transition has been derived in terms of $\xi(v \cdot v')$. These symmetries are then broken by perturbative effects. The perturbative corrections to symmetry-breaking corrections to first order in the strong coupling α_s are then computed at large recoil regime.

Zusammenfassung

Das Standard-Modell der Teilchenphysik liefert nur eine Parametrisierung der 'Flavor'-Eigenschaften; darin kommen die Quark- und Lepton-Massen vor und eine unitäre Mischungs-Matrix, die CKM (Cabibbo-Kobayashi-Masakawa) Matrix. In dieser Arbeit wird das Matrixelement $|V_{cb}|$ untersucht, insbesondere die Frage möglicher rechtshändiger Beimischungen zusätzlich zu den üblichen linkshändigen schwachen Strömen.

Ein links-rechts-symmetrisches Modell (LRSM) kann die Grundlage für die Untersuchung auch rechtshändiger Ströme sein. In der Literatur wurde es jedoch bisher nicht unter Einschluss von Flavor betrachtet. Im ersten Teil dieser Dissertation wird eine zusätzliche Flavor-Symmetrie in ein LRSM eingefügt und so die Untersuchung von Flavor ermöglicht. Der zweite Teil der Arbeit befasst sich mit den praktischen Fragen der Bestimmung eines rechtshändigen Beitrages. Der Ausgangspunkt der Untersuchungen ist der grundlegende $b \to c$ Übergang, dessen Händigkeit mit Hilfe von "Heavy Quark Symmetrien" (HQS) experimentell untersucht werden kann. Konkret werden in dieser Arbeit die Zerfälle $B \to D(D^*)\ell\bar{\nu}_\ell$ untersucht, die in der Literatur im Detail bereits für maximalen Impulsübertrag auf die Leptonen untersucht worden sind. In der vorliegenden Arbeit werden diese Zerfälle im Bereich des kleinen Impulsübertrages auf die Leptonen betrachtet, wobei der Formalismus der "Soft Collinear Effective Theory" (SCET) benutzt wird. Es wird eine Faktorisierungs-

formel hergeleitet, die in führender Ordnung das bekannte Resultat, nämlich die Reduktion auf einen Formfaktor $\xi(v \cdot v')$, liefert. Zu dieser Faktorisierungsformel werden die harten Streubeträge berechnet, indem die störungstheoretischen, symmetriebrechenden Korrekturen berechnet werden. Mit diesen Ergebnissen kann nunmehr der gesamte Phasenraum für eine Analyse genutzt werden.

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Chapter 1

Introduction

All known phenomenology of elementary particles can be described in terms of Standard Model, which has turned out to be an extraordinarily successful theory. It describes all known phenomenology from very low scales up to scales of the order of a few hundred GeV, the highest currently accessible energies.
Basis for Standard Model is the electroweak gauge theory, which unifies electromagnetic and weak interactions. The electroweak Lagrangian can be formulated with just one lepton-quark family consisting of the electron and its neutrino together with up and down quarks. Two other lepton-quark families are established experimentally and they follow the same pattern as the first one, i.e. the quarks and leptons have the same electroweak quantum numbers. The reason for the triplication is yet unknown.

Electroweak symmetry is known to be broken and a mechanism of spontaneous symmetry breaking is assumed. In this process masses of quarks and leptons appears as Yukawa couplings to the scalar sector. Within the Standard Model the pattern of mixing between the families is completely determined by the Yukawa couplings, incorporating also CP-Violation in the case of more than two families. Understanding flavor mixing in the quark and the leptonic sectors and the mechanism needed to break the electroweak symmetry are most important problems of high energy physics.

However, within the Standard Model flavor mixing is only parametrized in terms of the Yukawa couplings, inducing a large number of parameters. Together with lepton mixing, there are are a total of 26 independent parameters in Standard Model. With only 3 gauge parameters (the strong coupling constant α_s, the electromagnetic coupling α_{em} and the weak mixing angle Θ_W) remaining 23 parameters are originating from the less understood symmetry breaking sector of Standard Model. Reduction of the number of Standard Model parameters needs physics beyond the Standard Model. However, prediction of the angles and phases of the CKM matrix needs additional input such as symmetries between families, so-called horizontal symmetries.

Over last ten years, the elements of the CKM matrix have been measured experimentally. Experiments at the B-factories are producing a lot of information about decays of B-mesons, providing means to directly measure the elements of the CKM matrix with high statistics and low systematic errors. As CKM elements are fundamental parameters, they should be measured as precisely as possible. In future, when new particles are observed at the LHC, it will be important to know precise values for flavor parameters to understand the underlying physics. The over-constraining measurements of CP asymmetries, mixing, semileptonic, and rare decays have started to severely constrain the magnitudes and phases of possible new physics contributions to flavor-changing interactions .

Many ideas for physics beyond the Standard Model have been actively pursued by theorists for decades. SuperSymmetry (SUSY), theory of extra dimensions, non-commutative space time geometry, little Higgs models etc. are few of the candidates for new physics. However, most of them are designed to cure the hierarchy problem between the electroweak and the Planck scale and do not any theoretical insight into the flavor problem. As an alternative generic methods like effective field theories are developed for parametrizing new physics including the flavor sector. These methods are well suited to problems involving widely disparate mass scales and hence can even be applied to investigations reaching beyond Standard Model.

Precise determinations of Standard Model flavor parameters require to study interactions among quarks. On the other hand, experiments involve hadrons in which the quarks are bound by the strong force, QCD. Hence a determination of these fundamental parameters involves scales as high as the weak scales defined by the weak-boson mass and as low as Λ_{QCD}, the scale defined by strong interactions binding the quarks into hadrons. Thus this is an ideal field of application of effective theories. The theory of weak interactions seen at low scales of weak decays of hadrons is an effective theory ($m_{Hadron} \ll M_W$), as is the effective theory for heavy quarks ($\Lambda_{QCD} \ll m_{Quark}$) and the chiral limit of QCD ($m_\pi \ll \Lambda_{\chi\mathrm{SB}}$). Recently lepton mixing has also started to become an interesting subject along with quark mixing, since neutrino oscillations and thus also neutrino masses seem to have been established by recent experiments. However lepton mixing has not been discussed here.

The main idea of this dissertation is the study of CKM matrix and its elements using symmetries. As stated earlier the predictions of angles and phases of CKM matrix needs some additional inputs so the first portion of the thesis deals with the construction of a new model by extending the existing Left-Right Symmetric Models (LRSMs) by applying an additional $U(1)_{family}$ symmetry. LRSMs not only give the natural basis to present neutrinos as Majorana particles thus giving them mass with the help of seesaw mechanism but also offer additional sources of CP-violation coming from the right-handed Cabibo-Kobayashi-Maskawa(CKM) matrices as well as from the extended Higgs sector of the theory. Another important consequence of LRSMs is the appearance of right handed currents along with left handed ones, allowing for a test in heavy quark decays with the help of heavy quark effective theory due to the presence of heavy quark symmetry.

Right handed currents obtained form models like LRSMs can be studied with the help of heavy quark symmetries in effective theories. These calculations can can be used for the helicity measurements of $b \to c$ couplings. These measurements can be made by the study of semileptonic decays either inclusively $B \to X_c l \bar{\nu}$ or exclusively $B \to D(D^*) l \bar{\nu}$. A lot of work has been done in inclusive semileptonic decays as they give the value of $|V_{cb}|$ with low errors. However, hopes are high to obtain information on possible right-handed coupling through exclusive decays

measurements due to enriched nature of the accessible final states.
In exclusive decays determination of $|V_{cb}|$ has to rely on calculation of relevant form factors based on their normalization at $v = v'$, i.e. non recoil point, given by heavy quark symmetry. Any deviations from this heavy quark limit gives the value for V_{cb}. However a complementary case $v \cdot v' \sim \mathcal{O}\sqrt{m_b/m_c}$, i.e. maximum recoil point, is also of great interest. Precise theoretical predictions for such high-energy processes relies on the concept of factorization which systematically separates the short and long distance physics. soft Collinear Effective theories provides ans elegant method to separate the long and short distance physics by making use of collinear charm quark. Once factorization is done long distance physics can be studied with the help of hadronic form factors of composite operators. Charles et al have shown that when the momentum of final state meson is large certain symmetries are applied to these hadronic form factors. These symmetries reduces the number of independent form factors, but they are broken by radiative corrections.

On experimental side $BELLE$ and $BABAR$ have performed a detailed examination of both the $B \to D$ and $B \to D^*$ exclusive semileptonic modes. The $B \to D$ transition is governed by the vector current only and hence the corresponding hadronic matrix element is quite simple. A more complex and interesting pattern occurs for the $B \to D^*$ case, where analysis sought to extract form factor information and, in particular to measure the forward-backward asymmetry of the charged lepton, the average D^* polarization as well as $|V_{cb}|$.
Recently constraints on right-handed admixture to the weak $b \to c$ current from semileptonic decays are calculated which shows that tensions between the exclusive and inclusive determinations of $|V_{cb}|$ has been soften by considering the presence of right handed admixtures in weak hadronic currents.

Chapter 2

Right-Handed Currents

In spite of all the successes of the Standard Model (SM), it is unlikely to be the final theory. It leaves many unanswered questions, specially about the origin of quark and lepton masses and the hierarchy of family masses, quark mixing angles. Perhaps if the above questions are understood, the origin to CP-violation, the solution to the strong CP problem are also known.

2.1 Basics of Standard Model

The standard model is a relativistic quantum field theory that describes all know interactions of quarks and leptons [1]–[5]. To date, almost all experimental tests agreed with the predictions of standard Model. The Standard Model (SM) is constructed as a spontaneously broken $SU(3)_{color} \times SU(2)_W \times U(1)_Y$ gauge theory, where the $SU(3)_{color}$ corresponds to the strong interaction and the $SU(2)_W \times U(1)_Y$ induces the electroweak interaction.
The fundamental particles are ordered into three families of fermions having spin 1/2. The gauge group has 12 generators, corresponding to eight gluons g, three weak bosons $W^{\pm}$ and Z^0, and the photon mediating the electromagnetic interactions. The left-handed fermions are grouped into doublets of $SU(2)$ in the following way:

$$L_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}, \qquad Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \tag{2.1.1}$$

The right-handed fermions are $SU(2)$ singlet given as

$$e_{iR}, \qquad u_{iR}, \qquad d_{iR} \tag{2.1.2}$$

L_{iL},e_{iR} are the lepton fields and Q_{iL},u_{iR},d_{iR} are the quark fields. The subscript L and R means the left- and right-handed fields given as

$$\psi_L = \frac{1}{2}(1 - \gamma_5) \qquad \psi_R = \frac{1}{2}(1 + \gamma_5) \tag{2.1.3}$$

and index i is a family or generation index with i = 1,2,3.

2.1.1 The electroweak interaction and Higgs mechanism

The unification of electromagnetic and weak interaction to so-called electroweak interaction is given by *Glashow*, *Weinberg* and *Salam* [6][7][8]. There theory unifies the $SU(2)_L$ gauge group with the $U(1)_Q$ gauge group to the $SU(2)_L \times U(1)_Y$ electroweak gauge group. If the fundamental gauge fields of $SU(2)_L$ are W_1,W_2 and W_3 and of $U(1)_Y$ is B then the physical fields of electroweak interaction are given by

$$\begin{aligned} W^{\pm} &= \frac{W^1 \mp iW^2}{\sqrt{2}} \\ Z &= \cos\theta_W W^3 - \sin\theta_W B, \\ A &= \sin\theta_W W^3 - \cos\theta_W B \end{aligned} \tag{2.1.4}$$

where θ_W is the Weinberg angle having the value $\sin^2\theta_W \approx 0.231$ [9]. The Lagrangian's electroweak interaction term $\mathcal{L}_{EW}$ between the gauge bosons and the fermions is divided into charged current part $\mathcal{L}_{CC}$ and a neutral current part $\mathcal{L}_{NC}$. The charge current part is divided into leptonic part and the quark part,

$$\mathcal{L}^q_{CC} = -\frac{1}{\sqrt{2}} g \left(\bar{u}'_{Li} \gamma^\mu d'_{Li} W^+_\mu + \bar{d}'_{Li} \gamma^\mu u'_{Li} W^-_\mu \right) \tag{2.1.5}$$

$$\mathcal{L}^l_{CC} = -\frac{1}{\sqrt{2}} g \left(\bar{\nu}_{Li} \gamma^\mu e_{Li} W^+_\mu + \bar{e}_{Li} \gamma^\mu \nu_{Li} W^-_\mu \right). \tag{2.1.6}$$

where g is the weak coupling constant, The prime ($'$) denotes a quark state to be the eigenstate of the electroweak interaction (in contrast to the mass eigenstate state).

A very important part of SM concerns the masses of fermions and bosons. Higgs mechanism [10] is used to get mass term is the SM Lagrangian. As addition of mass term by hand will destroy the local $SU(2)$ gauge invariance. The method is to introduce a scalar, weakly interacting Higgs-doublet

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \tag{2.1.7}$$

with a potential allowing symmetry to break spontaneously. This Higgs when interact with fermions gives rise to Yukawa terms. These Yukawa terms allow flavor changing interactions and CP violation.

The Higgs term in SM-Lagrangian is written as

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - V(\Phi), \tag{2.1.8}$$

where D_μ is the covariant derivative and $V(\Phi)$ is the Higgs potential

$$V(H) = -\mu^2 \Phi^\dagger \Phi + \lambda^2 (\left(\Phi^\dagger \Phi\right)^2. \tag{2.1.9}$$

Due to $\mu^2 > 0$ and $\lambda^2 > 0$ the potential $V(\Phi)$ has a Mexican Hat shape resulting in a non-zero vacuum expectation value with

$$\sqrt{\Phi^\dagger \Phi} = \frac{\nu}{\sqrt{2}} = \frac{\mu}{\sqrt{2}\lambda} \tag{2.1.10}$$

for the ground state of Φ responsible for breakdown of $SU(3) \times SU(2) \times U(1)$ symmetry to $SU(2) \times U(1)_Q$.

2.1.2 The Yukawa Couplings and CKM matrix

$SU(3) \times SU(2) \times U(1)$ gauge invariance prevents bare mass terms for the quarks and leptons from appearing in the Lagrange density. The quarks and leptons not only get masses because of their Yukawa couplings to the Higgs doublet but also complete flavor structure of the SM is fixed because of them. The possible renormalizable interaction between the scalar fields and the quarks is

$$\mathcal{L}_{\text{Yukawa}} = -\Lambda^u_{ij}\bar{Q}'_{Li}\tilde{\Phi}u'_{Rj} - \Lambda^d_{ij}\bar{Q}'_{Li}\Phi d'_{Rj} - \Lambda^e_{ij}\bar{L}'_{Li}\Phi e'_{Rj} + h.c. \tag{2.1.11}$$

where Λ^u_{ij}, Λ^d_{ij} and Λ^e_{ij} are Yukawa coupling constants and are in general complex. They are the only source of the CP-violation in the SM. Since Φ has a vacuum expectation value, after spontaneous symmetry breaking the Yukawa couplings in Eq.(2.1.11) gives rise to 3×3 quark and lepton mass matrices. The Lagrangian in Eq.(2.1.11) thus gives

$$\mathcal{L}^q_{\text{Yukawa}} = -m^u_{ij}\bar{u}'_{Li}u'_{Rj} - m^d_{ij}\bar{d}'_{Li}d'_{Rj} + h.c. \tag{2.1.12}$$

where q stands for only quarks as only quark term from the Lagrangian has been written, with $m^u_{ij} = \Lambda^u_{ij}\frac{\nu}{\sqrt{2}}$ and $m^d_{ij} = \Lambda^d_{ij}\frac{\nu}{\sqrt{2}}$. Same Yukawa terms holds for leptons as well enabling them to get masses after spontaneous symmetry breaking. However, there is an exception for neutrinos due to the absence of right-handed neutrino field in SM they do not get mass from Yukawa interaction. By unitary transformation matrices V^u_L, V^u_R and V^d_L, V^d_R one can diagonalize the mass matrices m^u and m^d:

$$\begin{aligned}(V^u_L)^\dagger m^u (V^u_R) &= \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \\ (V^d_L)^\dagger m^d (V^d_R) &= \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}.\end{aligned} \tag{2.1.13}$$

The electroweak eigenstates $u'_i(d'_i)$ and the mass eigenstates $u_i(d_i)$ are related by

$$u'_{Li} = V^u_{Lij}u_{Lj} \quad d'_{Li} = V^d_{Lij}d_{Lj} \tag{2.1.14}$$

$$u'_{Ri} = V^u_{Rij}u_{Rj} \quad d'_{Ri} = V^d_{Rij}d_{Rj}. \tag{2.1.15}$$

The Eq.(2.1.5) in terms of mass eigenstates is then written as

$$\mathcal{L}^q_{CC} = -\frac{1}{\sqrt{2}}g\left(\bar{u}_{Li}\gamma^\mu V_{ij}d_{Lj}W^+_\mu + h.c\right). \tag{2.1.16}$$

where the matrix $(V_{ij}) = \mathbf{V}$ is defined as

$$\mathbf{V} = (V^u_L)^\dagger V^d_L. \tag{2.1.17}$$

and is known as Cabibbo-Kaboyashi-Maskawa matrix (CKM). The understanding of flavor dynamics, and of the related origin of quark and lepton masses and mixing, is among the most important goals in elementary particle physics. In this context, weak decays of hadrons, and in particular the CP violating and rare decay processes, play an important role as they are sensitive to short distance phenomena. Therefore the determination of the Cabibbo-Kobayashi-Maskawa matrix that parametrizes the weak charged currents interactions of quarks is currently a central theme in high energy physics.

The matrix $\mathbf{V}$ from Eq.(2.1.16) is a 3×3 unitary matrix and is given as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2.1.18}$$

with V_{ud} specifying flavor mixing of u and d quarks.
The unitarity of the CKM matrix can be expressed by the relation

$$\sum_{k=1}^{3} V_{ki} V_{kj}^{*} = \delta_{ij} \qquad \text{for } i,j = 1,2,3 \tag{2.1.19}$$

Being a unitary matrix it is completely specified by nine real parameters which can be implemented as three mixing angles and six phases. The six phases can then be chosen in such a way that it eliminates five out of six phase parameters. As a result $\mathbf{V}$ can then be expressed in terms of three rotation angles and one phase.

There are infinite ways to express the elements of $\mathbf{V}$, one representation has been sanctioned by the particle data group [9] given as

$$\mathbf{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ -s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.1.20}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for the mixing angles θ_{ij} and δ known to be Kobayashi-Maskawa-phase.

Without some specific ideas about the mechanism for the flavor generation, none of the representation of $\mathbf{V}$ is superior for theoretical reason, but some can be more convenient on phenomenological grounds one of which is the Wolfenstein representation [11]. Wolfenstein pointed out that CKM matrix can also be expressed through the expansion in powers of $\sin\theta_{12} = \lambda$. The three mixing angles and one KM phase of the above representation are replaced by four real quantities λ, A, ρ and η. It is given as

$$\mathbf{V} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & a\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{2.1.21}$$

The definition of the parameter set (λ, A, ρ, η) is divided into one part defining the order of magnitude scale for the matrix elements (λ) and into another part defining the fine tuning parameters for the matrix elements (A, ρ, η). A and $\sqrt{\rho^2 + \eta^2}$ are of order 1. From the numbers of powers in λ one easily gets the order of magnitude of any CKM term.
A further advantage of the definition of (λ, A, ρ, η) is that the Wolfenstein approximation can be improved to be accurate up to $\mathcal{O}(\lambda^2)$ [12]. In this case a parameter

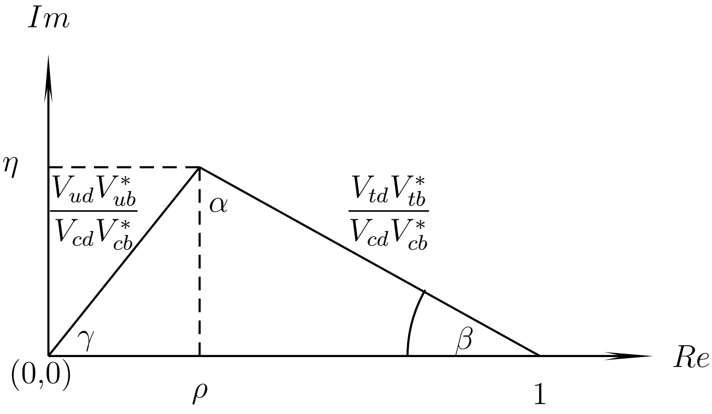


Figure 2.1: Unitarity Triangle.

set (λ, A, $\bar{\rho}$, $\bar{\eta}$) with $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$ is used to parametrize $\mathbf{V}$. The approximate values of the CKM matrix parameters are $\lambda \approx 0.227$, $A \approx 0.818$, $\bar{\rho} \approx 0.221$ and $\bar{\eta} \approx 0.340$[9].

Unitarity relation given in Eq.(2.1.19) for $i \neq j$ gives three equations, each summing up three complex numbers to zero. The three equations can be represented by three triangles in the complex plane. Using Wolfenstein parametrization one can easily see by counting of λ-powers that two equations sum up terms of different magnitude ($i = 1,3; j = 2$) whereas one sums up terms of comparable magnitude ($i = 1; j = 3$). The letter case is a non-compressed notation reads

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{2.1.22}$$

After the division of this equation by its second term one obtains

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \tag{2.1.23}$$

The Eq.(2.1.23) can be expressed in the form of triangle and the corresponding triangle is shown in Fig(2.1) and is called as unitarity triangle. It depends only on CKM parameters ρ and η. The angles of the triangle are given by

$$\begin{aligned} \alpha &= arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \\ \beta &= arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \\ \gamma &= arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] = \pi - \alpha - \beta \end{aligned} \tag{2.1.24}$$

and the side lengths of the triangle can be read from Eq.(2.1.23). A very important property of the unitarity triangle is that all of its side lengths and angles can be determined from B meson observations thus making the study of B physics very important.

2.2 Quantum Chromodynamics

The portion of SM that describes the strong interactions of quarks and gluons making up hadrons (such as proton, neutron and pion) is called quantum chromodynamics (QCD). It is the study of $SU(3)$ Yang-Mills theory of color-charged fermions (the quarks). The charge of strong interaction has color index red, green and blue (RGB). All quark flavors (u, d, c, s, t, b) are assigned with this color charge. All hadrons are color neutral, which means that their wave function has a linear combination of color states with either the same amplitudes of all three colors (baryons) or in the form of combination of color and anti color (mesons).
QCD enjoys two peculiar properties

- Asymptotic freedom: In QCD the coupling α_s, defined by QCD gauge coupling constant g_s as $\alpha_s = \frac{g_s^2}{4\pi}$, depends upon the momentum transfer Q^2: $\alpha_s(Q^2)$, decreases with growing Q^2 and in the limit $Q^2 \to \infty$ the coupling vanishes.
- Confinement: which means that the force between quarks does not diminish as they are separated. Because of this, it would take an infinite amount of energy to separate two quarks; they are forever bound into hadrons such as the proton and the neutron.

QCD is a non-abelian gauge theory, with the gauge group $SU(3)_C$, with 'C is denoted for color. The 8 generators of $SU(3)_C$ are represented by Gell-Mann matrices $T^a = \lambda_a/2$. A gauge transformation is of the form

$$Q'^{\alpha,q} = e^{-ig_s\theta^a(x)T^a} Q^{\alpha,q} \tag{2.2.1}$$

where g_s is the coupling constant. The QCD Lagrangian is

$$\mathcal{L}_{QCD} = \bar{Q}^{\alpha,q}\left(i\not{D} - m_q\right)Q^{\alpha,q} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} \tag{2.2.2}$$

The covariant derivative is written as

$$D_\mu = \partial_\mu + ig_s A^a_\mu T^a \tag{2.2.3}$$

with A^a_μ are the gauge bosons called as gluons. The field strength tensor of the gluon fields is

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \tag{2.2.4}$$

with the structure constants f^{abc} of the $SU(3)$:

$$\left[T^a, T^b\right] = if^{abc}T^c \tag{2.2.5}$$

The field strength can be contracted with the group generator ($A_\mu = A^a_\mu T^a$ and $G_{\mu\nu} = G^a_{\mu\nu}T^a$) and hence can be written as commutator of covariant derivatives

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s\left[A_\mu, A_\nu\right] = \frac{i}{g_s}\left[D_\mu, D_\nu\right] \tag{2.2.6}$$

The kinetic term for the gluon fields in $\mathcal{L}_{QCD}$ can be rewritten as

$$\mathcal{L}_{QCD} = \bar{Q}^{\alpha,q}\left(i\not{D} - m_q\right)Q^{\alpha,q} - \frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right]. \tag{2.2.7}$$

This non-abelian structure in the kinetic term of the Lagrangian amounts for the self-coupling of the gluons giving the evidence that they carry color charge.

The understanding of the connection between quarks and hadrons properties is a prerequisite for a precise determination of the parameters of the SM and often the theoretical description of hadron properties relies on very naive bound state models with no direct connection to the complete QCD. This hence provides the idea of approximating the theory according to the particular system under consideration on the basis of energy scale, which can always then be improved by taking into account the corrections induced by the neglected energy scales as small perturbations. Chiral Perturbation Theory (ChPT) is the low-energy realization of QCD in the light quark sector is one of such example where masses of the light quark (u, d, s) can be set to zero. Similarly Heavy Quark Effective Theory (HQET) is a QCD approximation with masses of heavy quarks (c, t, b) are taken to be infinitely large. Both of the above approximations leads to different sets of symmetries on the basis of which reliable predictions can be made. HQET is discussed and explained in next chapter to calculate the symmetry breaking corrections to $\bar{B} \to D^{(}D^{*})l\bar{\nu}$ decay at large recoil limit.

2.3 Physics beyond the SM

Even though the Standard Model remains unchallenged by an impressive body of precision electroweak measurements, it does have some features which are considered as unsatisfactory by the physicists. As at one hand, there exist no understanding of the number of families, the origin of quantum number assignments or the large number of arbitrary parameters. On the other hand, there are difficulties in obtaining small enough values of the cosmological constant, the strong CP-violating parameter, the quadratic radiative corrections to the mass of Higgs boson. One of very important question is smallness of the off-diagonal matrix elements of CKM matrix and the absence of its leptonic counter part MNS. It is thus necessary to look beyond the Standard Model to get answers to these unresolved questions.

Theoretical ideas about physics beyond the Standard Model have been strongly influenced by the success of gauge theories. For example, Grand Unified Theories (GUTs) have been considered as a natural extension of the Standard Model, but GUTs still simply triplicate the particle content to take into account the three families. Except for the fact that leptons and quarks are members of the same multiplet of the group used for grand unification, and thus their masses have to be equal at the GUT scale, any ansatz that might answer all or at least some of the above questions, is unable to be obtained from these theories.

Other important theories to look for the physics beyond SM is the SuperSymmetric theories (SUSY). These theories have been widely used to explain the high-energy-physics data. As far as the symmetry-breaking sector and the mixing of flavors are concerned, the situation in a supersymmetric theory becomes much worse than in the Standard Model. Aside from the fact that the three generations are introduced by hand just as in the Standard Model, the Higgs sector needs to be extended in order to comply with supersymmetry. Furthermore, the introduction of the supersymmetric partners of the existing SM particles yields many more sources of flavor mixing and CP violation, and a serious fine tuning (or some other special assumption) is needed for the theory to be consistent with data. In particular, the observed small CP violation, appearing only in the charged-current sector, cannot be introduced into a supersymmetric theory in a natural way. On the basis of current knowledge, it is fair to say that supersymmetry clearly has a flavor problem.

Looking for physics beyond SM one idea is to simply extend existing SM without changing its structure. There are many ideas discussed in literature about how to do extension, one of them is Left-Right Symmetric Models (LRSMs)[13][14][15].LRSMs are based on the fact that right handed doublets should also be present along with the left handed ones thus giving the SM so called left-right symmetry. The most amazing feature of the LRSMs is the presence of the right-handed neutrinos having Yukawa couplings, which have interesting possibility of generating small neutrino masses through see-saw mechanism. Another importance of LRSMs is that they can be merged together in the context of grand unified schemes.

2.4 Left-Right Symmetric Models (LRSMs)

Basic Standard Model Lagrangian lacks the Left-Right symmetry as only left handed neutrinos are introduced without any better explanation then the phenomenological fact that neutrinos are mass less or extremely light. Left Right Symmetric Models are based on the gauge group $SU(3)_C \times SU(2)_L \times \; SU(2)_R \times \; U(1)_{B-L}$, where left handed and right handed fermion fields are treated symmetrically. In such models Left-Right symmetry is broken at some high scale, yielding a parity-violating Standard Model-like theory at low energies.

The original left-right symmetric models based on an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. Because of the $L - R$ invariance the couplings of $SU(2)_L$ and $SU(2)_R$ are equal, i.e. $g_L = g_R = g$. The quarks are assigned in the following multiplets:

$$\begin{aligned} Q_{iL} &= \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \equiv (2{,}1{,}1/3) \\ Q_{iR} &= \begin{pmatrix} u_{iR} \\ d_{iR} \end{pmatrix} \equiv (2{,}1{,}1/3) \end{aligned} \tag{2.4.1}$$

For leptons one can write

$$\begin{aligned} L_{iL} &= \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} \equiv (2{,}1{,}-1) \\ L_{iR} &= \begin{pmatrix} \nu_{iR} \\ e_{iR} \end{pmatrix} \equiv (1{,}2{,}-1) \end{aligned} \tag{2.4.2}$$

withe i = 1,2,3 is the generation index, and the representation with respect to the gauge group is explicitly given.

The gauge bosons consists of two triplets:

$$\begin{aligned} \mathbf{W}_{\mu L} &= \begin{pmatrix} W^+_{\mu L} \\ Z^0_{\mu L} \\ W^-_{\mu L} \end{pmatrix} \equiv (3,1,0) \\ \mathbf{W}_{\mu R} &= \begin{pmatrix} W^+_{\mu R} \\ Z^0_{\mu R} \\ W^-_{\mu R} \end{pmatrix} \equiv (1,3,0) \end{aligned} \tag{2.4.3}$$

and one singlet

$$\mathbf{B}_\mu = B^0_\mu \equiv (1,1,0) \tag{2.4.4}$$

When the SM is extended to the LRSMs, one has the group of symmetries where the hypercharge quantum number Y now becomes $B-L$, the difference between the baryonic number B and the leptonic number L [20] [22]. Hence the electric charge in LRSMs is defined as

$$Q = I_{3L} + I_{3R} + \frac{(B-L)}{2} \tag{2.4.5}$$

In LRSMs $(B-L)$ is the gauge symmetry. At $E > M_W$, Q and I_{3L} are conserved, so parity and the local $B-L$ in-variances are broken spontaneously at the same time. Only a linear combination, $U(1)_Y$, of I_{3R} and $B-L$ remains unbroken. The symmetry breaking pattern can be written as:

$$\begin{gathered} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ \downarrow M_{W_R} \\ SU(3)_C \times SU(2)_L \times U(1)_Y \\ \downarrow M_{W_L} \\ SU(3)_C \times U(1)_{e.m} \end{gathered} \tag{2.4.6}$$

One of the Important issue in the LR models has been the scale at which right handed current interactions become significant. The constraints on the righthanded W bosons mass M_{W_R} has been explored in Ref. [57]. The most well-known and significant lower bound on M_{W_R} comes from the mass difference between K_L and K_S, in which the $W_L - W_R$ box-diagram contribution is enhanced by both the Wilson coefficient and hadronic matrix element, altogether by a factor of $\mathcal{O}(10^3)$. A re-evaluation of this contribution with updated values of strange quark mass and hadronic matrix element and consideration of CP violation observables yield a lower bound $M_{W_R} > 4.0$ TeV. The current experimental bound on M_{W_R} in direct collider search is about 800 GeV[9]. Given these values, it is possible to have a right handed gauge boson with mass of order $1-2$ TeV.

Physics of CP violation in LRSMs is quite interesting. The strong CP problem is solved by eliminating the dimension-four gluon operator $G\tilde{G}$ by parity at high energy. In the weak sector, two special scenarios have been generally discussed historically. One is called “manifest” LR symmetry without spontaneous CP violation, in which parity guarantees left-handed and right-handed CKM matrices are identical $V^R_{CKM} = V^L_{CKM}$. CP violation enters through the CKM matrix only. Moreover If $\langle H \rangle$ remains real, the up- and down-type quark mass matrices are Hermitian. The other is called " pseudo-manifest" LR symmetry such that the Lagrangian is invariant under P and CP, both of which are broken spontaneously, severely constraints the Yukawa couplings. CP violation then arises through the vevs of scalar fields acquiring a complex phase. V^R_{CKM} is then proportional to the complex conjugate of the V^L_{CKM} multiplied by additional phases, determined by those of the Higgs vacuum expectation values (vev) i.e $V^R_{CKM} = V^{*L}_{CKM} K$ K being the diagonal phase matrix. Despite two different assumptions for the origin of CP violation, here $|V^L_{CKM}| = |V^R_{CKM}|$ is considered to make discussion to be the simple.

Extension of gauge sector shows that one needs to extend the number of scalar particle to get required pattern of spontaneous breakdown of symmetry. First of all as both quarks and leptons are placed in doublets, one needs a bidoublet of scalar bosons compared to doublet in SM to implement the symmetry breaking mechanism

$$H \sim (2,2,0) \sim \begin{pmatrix} h_1^0 & h_1^+ \\ h_2^- & h_2^0 \end{pmatrix}, \qquad \tilde{H} = \tau_2 \, H^* \tau_2 \sim (2,2,0), \tag{2.4.7}$$

which is left-right symmetric and contains the SM Higgs doublet. However to arrive at the phenomenologically required symmetry pattern $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SU(3)_C \times SU(2)_L \times U(1)_Y$ and then to $SU(3)_C \times U(1)_{e.m}$ one needs additional Higgs multiplets. The simplest possibility is to choose a doublet [14]–[17].

$$\chi_R \sim (1,2,1) \sim \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \tag{2.4.8}$$

In order to keep the left-right symmetry one then needs to induce a left handed doublet as well

$$\chi_L \sim (2,1,1) \sim \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} \tag{2.4.9}$$

The $SU(2)_R$ is broken at the right handed scale M_{W_R} through the vevs of χ_R and electroweak symmetry is broken by vacuum expectation values (vev's) of H. As the H bidoublet leads to the same masses for the left-weak and the right-weak bosons with equal electric charge so it is worth full to introduce χ_R.

The Higgs sector can be extended by introducing two more triplets given as

$$\Delta_L \sim (3,1,2) \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & \frac{-\delta_L^+}{\sqrt{2}} \end{pmatrix}, \qquad \Delta_R \sim (1,3,2) \sim \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & \frac{-\delta_R^+}{\sqrt{2}} \end{pmatrix} \tag{2.4.10}$$

One of significance of the introduction of two triplets is as the doublets χ_L and χ_R are singlets of either $SU(2)_R$ or $SU(2)_L$ and so cannot interact in a re-normalizable way with the usual quarks and leptons. This mean that neutrinos are Dirac particles and can get their masses in the same way as other fermions making it difficult to understand the smallness of their masses. Two Higgs triplets (first suggested in [18]) is very useful to explain it as leptons can then interact with the Higgs triplets through the Majorana like Yukawa couplings. As a result neutrinos become Majorana particles and the see-saw [18][24][25] mechanism is operative. This mechanism provides a very natural explanation of the smallness of the usual neutrino masses, relating it to the fact that the right-handed scale is much higher than the electro-weak scale. Invariance under the discrete transformation is required such that

$$Q_L \leftrightarrow Q_R, \qquad H \leftrightarrow H^\dagger, \tag{2.4.11}$$

$$\Delta_L \leftrightarrow \Delta_R \qquad \chi_L \leftrightarrow \chi_R, \tag{2.4.12}$$

which means that $g_L = g_R$ as stated earlier. This may be softly broken by dimension - 2 operators in the Higgs potential. It is now well established that [19] [13] if the Higgs potential is exactly symmetric with respect to the discrete parity transformation, the vacuum can be chosen ($v_R \gg v_L$) in such a way that it would give a very elegant explanation of parity violation at low energies as being a result of spontaneous breakdown of symmetry. In other words, parity non-conservation at low energies and the smallness of neutrino masses have common origin in left-right symmetric model.

It is worth mentioning here that in most of the literature about LRSMs in order to break symmetry spontaneously two cases has been discussed: One in which one Higgs bi-doublet H and two Higgs triplet Δ_L, Δ_R are considered, and in 2nd choice two Higgs doublets χ_L, χ_R are taken along with bi-doublet H. Both these choices for scalar Higgs have their own significance depending upon the problem under consideration. However form of LRSMs under discussion here makes use of one Higgs bi-doublet H , two Higgs doublets χ_L, χ_R and two Higgs triplet Δ_L , Δ_R. The significance of using so many scalars will be clear in next section where a special case of left-right symmetric model is considered with an additional family symmetry. Although presence of so many scalars makes it difficult to avoid the presence of FCNC at tree level (to reproduce the SM like structure) but symmetry breaking scales can be adjusted to avoid them.

The most general scalar potential can be written as

$$V = V_H + V_\chi + V_\Delta + V_{H\chi} + V_{H\Delta} + V_{\chi\Delta} + V_{H\chi\Delta} \tag{2.4.13}$$

The discussion of the corresponding potential for the spontaneous symmetry breaking is beyond the goal of discussion. A very comprehensive discussion on the potential consistent with renormalizability, gauge invariance and discrete left-right symmetry is given in [13] (considering only one bi-doublet and two doublets). The consequence of the discrete left-right symmetry is that all the terms in the potential will be self conjugate, therefore all parameters must be real.
Consistent with the minima of the Higgs potential the vacuum expectation values of for the scalar field necessary to break the $L-R$ symmetry are given with the assumptions applied on them.

$$\langle H\rangle = \begin{pmatrix} v\sin\beta & 0 \\ 0 & v\cos\beta \end{pmatrix}, \qquad \langle\tilde{H}\rangle = \begin{pmatrix} v\cos\beta & 0 \\ 0 & v\sin\beta \end{pmatrix} \tag{2.4.14}$$

Similarly

$$\langle\chi_L\rangle = \begin{pmatrix} 0 \\ \lambda_L \end{pmatrix}, \qquad \langle\chi_R\rangle = \begin{pmatrix} 0 \\ \lambda_R \end{pmatrix} \tag{2.4.15}$$

and

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \qquad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \tag{2.4.16}$$

v is the Standard Model Higgs vacuum expectation values, since v_L breaks $SU(2)_L$ which must be preserved in the first symmetry breaking step so one must have to choose $v_R \gg v_L$ to give really heavy masses to the right-weak boson W_R^+, W_L^- and Z_R^0. [21] purposes that v_R must be at least 2.7×10^7GeV to match the experimental constraints coming from neutrinos. Also as Δ_L is an $SU(2)_L$ triplet so v_L must be smaller than v not to spoil the well known experimental condition $M_{W_L}^2/M_{Z_L}^2 \simeq \cos^2\theta_W$, $\rho = 1$ relation. Similarly $\lambda_L\lambda_R = \mathcal{O}(v^2)$ and $\tan\beta \gg 1$ is considered in order to explain the difference between the top and bottom masses.

2.5 LRSMs with additional $U(1)_{family}$ symmetry

One of the main reasons to go beyond SM is to get some insight about the hierarchy among the masses of different quark families. Here a family symmetry is imposed on the LRSMs to get some insight of flavor physics. However till now there is neither a generally accepted nor a predictive framework for flavor. The family symmetry applied has to satisfy certain constraints. The general assumption is that such a symmetry gives the observed structure to the quark mass matrices. One of the important fact of this symmetry is that it cannot be exact, so it has to be broken. Also another observation is that this symmetry cannot be broken spontaneously by the vacuum expectation value of the single Higgs field, hence the scalar sector needs to be extended. This has already be explained in the previous section taking into account more than one scalar Higgs namely, one Higgs bi-doublet, two doublet and one triplet fields.

Quark multiplets and scalar fields are considered to transforms under an additional family-dependent global $U(1)$ symmetry thus extending the gauge group as $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{family}$. With the requirement of reasonably small values for the new charges one can get a possible patterns leading to particular textures for the up and down type quark masses.

Using the appropriate $U(1)_{family}$ charges, aim is to allow or forbid certain entries in Yukawa matrices so that observed hierarchy in the quark mass matrices and CKM elements is obtained. Before getting the values for $U(1)_{family}$ it is better to look at the power counting of CKM matrix in terms of Wolfenstein parameters.

Quark masses and their approximate scaling with the Wolfenstein parameters $\lambda \sim \epsilon$ are quoted in Table(2.1). As stated earlier, in LRSMs right handed quarks are doublets under the $SU(2)_R$ gauge group, so there is a corresponding right handed CKM matrix V_{CKM}^R, analogous to the usual Standard Model CKM matrix V_{CKM}^L. As left-right symmetric model is set up with axial $U(1)_{family}$ charges, one expect the left- and right-handed CKM matrices to follow the same power counting i.e. $V_{CKM}^R = V_{CKM}^L$

$$V_{CKM}^R \sim V_{CKM}^L \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \tag{2.5.1}$$

	u	d	s
m_q	$[1.5-3.3]MeV$	$[3.5-6.0]MeV$	$[70-135]MeV$
$log_\lambda(m_q/m_t)$	$7-8$	$6-7$	$4-5$
	c	b	t
m_q	$[1.16-1.34]GeV$	$[4.13-4.37]MeV$	$\sim 172GeV$
$log_\lambda(m_q/m_t)$	$3-4$	$2-3$	1

Table 2.1: SM values for quark masses and approximate scaling with the Wolfenstein parameter $\lambda \sim 0.2$. Light quark masses (u,d,s) are given in the $\bar{MS}$ scheme at $\mu = 2GeV$ charm and bottom masses as $\bar{m_c}$ and $\bar{m_c}$ and top mass is evolved down to the scale m_b. The evaluation between the scales m_b and m_c is negligible for our considerations.

The power counting used for effective SM Yukawa matrices and CKM elements for up-type quarks is

$$\begin{aligned} Y_U &= V_L \text{diag}[y_u,y_c,y_t] V_R^\dagger \\ &\sim V_L \text{diag}[\epsilon^7,\epsilon^3,1] V_R^\dagger \\ &\sim \begin{pmatrix} \epsilon^7 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \end{aligned} \tag{2.5.2}$$

for down-type quarks one writes

$$\begin{aligned} Y_D &= \tilde{V}_L \text{diag}[y_u,y_c,y_t] \tilde{V}_R^\dagger \\ &\sim \tilde{V}_L \text{diag}[\epsilon^7,\epsilon^3,1] \tilde{V}_R^\dagger \\ &\sim \begin{pmatrix} \epsilon^{6(7)} & \epsilon^{5(6)} & \epsilon^5 \\ \epsilon^{5(6)} & \epsilon^{4(5)} & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \end{pmatrix} \end{aligned} \tag{2.5.3}$$

where its assumed that generically not only $V_{CKM} \sim V_L \sim V_R$ but also some elements of Y_U and Y_D could actually be smaller then denoted or even can be zero.

2.5.1 Possible Gauge Couplings

As Yukawa terms are responsible for giving masses to the quarks and the possible scalar couplings to fermion fields is strongly constrained by the $Left-Right$ gauge symmetry. So all possible operators satisfying the $Left-Right$ symmetry upto dimension-8 are considered. They are discussed one by one as under

Dimension-4 The dimension-4 terms are obtained by the coupling of quarks with the Higgs bi-doublet H and $\tilde{H}$

$$\begin{aligned} O^{(4)} &= (\bar{Q}_L H Q_R) \to v \sin\beta(\bar{U}_L U_R) + v\cos\beta(\bar{D}_R D_R), \\ \tilde{O}^{(4)} &= (\bar{Q}_L \, \tilde{H} \, Q_R) \to v\cos\beta(\bar{U}_L U_R) + v\sin\beta(\bar{D}_R D_R). \end{aligned} \tag{2.5.4}$$

with $\tilde{H} = \tau_2 H^* \tau_2$ .These contributions to up- and down-type Yukawa couplings are distinguished by $\tan\beta \gg 1$. Considering the possible power counting one can write

$$\sin\beta = \mathcal{O}(1), \qquad \cos\beta = \mathcal{O}(\epsilon^2), \tag{2.5.5}$$

such that one can generate the 3-3 elements of Y_U and Y_D (i.e. $m_t = v\sin\beta$ and $m_b = v\cos\beta$) from the operator $\mathcal{O}_4$, while the operator $\tilde{\mathcal{O}}_4$ has to be forbidden.

Dimension-5 Dimension-5 terms are obtained by the coupling of doublets χ_l and χ_R.

$$
\begin{aligned}
O^{(5)} &= \frac{1}{\Lambda}(\bar{Q}_L\chi_L^c)(\chi_R^T Q_R) \to v\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{U}_L U_R) \\
\tilde{O}^(5) &= \frac{1}{\Lambda}(\bar{Q}_L\chi_L)(\chi_R^\dagger Q_R) \to v\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{D}_L D_R)
\end{aligned}
\tag{2.5.6}
$$

Here the definition of the transposed fields includes the anti-symmetric tensor in $SU(2)$, i.e. $(\chi_R^T\, Q_R) \equiv \epsilon_{ij}\, \chi_R^i\, Q_R^j$. It is to be noticed that the dimension-5 contributions to the quark mass matrix are all suppressed by the $\mathcal{O}(v/\Lambda)$.

It is assumed that the ϵ^2 contribution to the 2-3 element of Y_U arises from the dim-5 terms which fixes

$$
\frac{v}{\Lambda} \sim \mathcal{O}(\epsilon^2)
\tag{2.5.7}
$$

and it can be further suggested

$$
\frac{v_R}{\Lambda} \sim \frac{\lambda_R}{\Lambda} \sim \mathcal{O}(\epsilon), \qquad \frac{v_L}{\Lambda} \sim \frac{\lambda_L}{\Lambda} \sim \mathcal{O}(\epsilon^3)
\tag{2.5.8}
$$

with this we have

$$
\mathcal{O}_5 \sim \tilde{\mathcal{O}}_5 \sim \epsilon^2
\tag{2.5.9}
$$

Dimension-6 Dimension-6 operators are given as under

$$
\begin{aligned}
O_a^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\chi_L^c)(\chi_R^\dagger\Delta_R Q_R) \to v\frac{v_R}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{U}_L U_R), \\
\tilde{O}_a^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\chi_L)(\chi_R^T\Delta_R^\dagger Q_R) \to v\frac{v_R}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{D}_L D_R), \\
O_b^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger\chi_L)(\chi_R^T Q_R) \to v\frac{v_L}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{U}_L U_R), \\
\tilde{O}_b^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L\chi_L^c)(\chi_R^\dagger Q_R) \to v\frac{v_L}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{D}_L D_R), \\
O_c^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger\tilde{H}\Delta_R Q_R) \to v\sin\beta\frac{v_L v_R}{\Lambda^2}(\bar{U}_L U_R), \\
\tilde{O}_c^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L H\Delta_R^\dagger Q_R) \to v\sin\beta\frac{v_L v_R}{\Lambda^2}(\bar{D}_L D_R), \\
O_d^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger H\Delta_R Q_R) \to v\cos\beta\frac{v_L v_R}{\Lambda^2}(\bar{U}_L U_R), \\
\tilde{O}_d^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L\tilde{H}\Delta_R^\dagger Q_R) \to v\sin\beta\frac{v_L v_R}{\Lambda^2}(\bar{D}_L D_R)
\end{aligned}
\tag{2.5.10}
$$

this is to be kept in mind that $\Delta_L = \Delta_L^a \tau^a$ and also $\Delta_R = \Delta_R^a \tau^a$. Along with the above mentioned dim-6 couplings one also gets

$$
\begin{aligned}
O_e^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L \Delta_L H Q_R)\mathrm{tr}[H^\dagger \tilde{H} + \tilde{H}^\dagger H] \\
&\to v\frac{v^2 \sin 2\beta}{\Lambda^2}(\sin\beta(\bar{U}_L U_R) + \cos\beta(\bar{D}_L D_R)), \\
\tilde{O}_e^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L \Delta_L \tilde{H} Q_R)\mathrm{tr}[H^\dagger \tilde{H} + \tilde{H}^\dagger H] \\
&\to v\frac{v^2 \sin 2\beta}{\Lambda^2}(\cos\beta\bar{(}U_L U_R) + \sin\beta(\bar{D}_L D_R))
\end{aligned}
\tag{2.5.11}
$$

There are also some other dim-6 operators which are irrelevant as they have same combinations of scalar quantum numbers as dim-4 and hence will not generate new entries in the quark Yukawa matrices. The power counting discussed above thus can be given as

$$
\begin{aligned}
\mathcal{O}_{6a} \sim \tilde{\mathcal{O}}_{6a} &\sim \epsilon^3 \\
\mathcal{O}_{6c} \sim \tilde{\mathcal{O}}_{6c} &\sim \epsilon^4 \\
\mathcal{O}_{6b} \sim \tilde{\mathcal{O}}_{6b} &\sim \epsilon^5 \\
\mathcal{O}_{6e} \sim \tilde{\mathcal{O}}_{6e} &\sim \epsilon^6 + \epsilon^8 \\
\mathcal{O}_{6d} &\sim \epsilon^6 \\
\tilde{\mathcal{O}}_{6d} &\sim \epsilon^4
\end{aligned}
\tag{2.5.12}
$$

Dimension-7 The following operators contribute into dimension-7

$$
\begin{aligned}
O_a^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L H Q_R)(\chi_R^\dagger \Delta_R \chi_R^c + \chi_L^T \Delta_L^\dagger \chi_L^c) \\
&\to v\frac{v_R\lambda_R^2 + v_L\lambda_L^2}{\Lambda^3}(\sin\beta(\bar{U}_L U_R) + \cos\beta(\bar{D}_L D_R)) \\
\tilde{O}_a^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L \tilde{H} Q_R)(\chi_R^\dagger \Delta_R \chi_R^c + \chi_L^T \Delta_L^\dagger \chi_L^c) \\
&\to v\frac{v_R\lambda_R^2 + v_L\lambda_L^2}{\Lambda^3}(\cos\beta(\bar{U}_L U_R) + \sin\beta(\bar{D}_L D_R)) \\
O_b^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L H Q_R)(\chi_L^c \Delta_R \chi_R^\dagger + \chi_L \chi_R^T \Delta_R^\dagger) \\
&\to v\frac{v_R\lambda_R\lambda_L}{\Lambda^3}(\sin\beta(\bar{U}_L U_R) + \cos\beta(\bar{D}_L D_R)) \\
\tilde{O}_b^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L \tilde{H} Q_R)(\chi_L^c \Delta_R \chi_R^\dagger + \chi_L \chi_R^T \Delta_R^\dagger) \\
&\to v\frac{v_R\lambda_R\lambda_L}{\Lambda^3}(\cos\beta(\bar{U}_L U_R) + \sin\beta(\bar{D}_L D_R))
\end{aligned}
\tag{2.5.13}
$$

which thus gives rise to terms in power counting as

$$
\begin{aligned}
&\mathcal{O}_{7a} \sim \tilde{\mathcal{O}}_{7a} \sim \epsilon^3 + \epsilon^5 + \text{higher order terms} \\
&\mathcal{O}_{7b} \sim \tilde{\mathcal{O}}_{7b} \sim \epsilon^5 + \epsilon^6
\end{aligned}
\tag{2.5.14}
$$

Dimension-8 Following are the contributions from dimension-8 operator

$$
\begin{aligned}
O_a^{(8)} &= \frac{1}{\Lambda^3}(\bar{Q}_L H H^\dagger Q_R)(\chi_L^c \Delta_R \chi_R^\dagger) \\
&\to v^2 \frac{v_R \lambda_L \lambda_R}{\Lambda^4}(\sin^2\beta \bar{U}_L U_R + \cos^2\beta \bar{D}_L D_R) \\
\tilde{O}_a^{(8)} &= \frac{1}{\Lambda^3}(\bar{Q}_L \tilde{H} \tilde{H}^\dagger Q_R)(\chi_L^c \Delta_R \chi_R^\dagger) \\
&\to v^2 \frac{v_R \lambda_L \lambda_R}{\Lambda^4}(\cos^2\beta \bar{U}_L U_R + \sin^2\beta \bar{D}_L D_R)
\end{aligned}
\tag{2.5.15}
$$

The power counting of the operators involved are

$$
\mathcal{O}_{8a} \sim \tilde{\mathcal{O}}_{8a} \sim \epsilon^7 + \text{higher order terms} \tag{2.5.16}
$$

Notice that in order to take into account all possible quark mass contributions to a given order ϵ^n, one have to consider operators up to dimension $d = 4 + n$. To keep the discussion simple we will not consider terms of the order ϵ^6 or higher.

2.5.2 Choosing appropriate $U(1)_{family}$ charges

Apart from the power counting specified in last section, in order to reproduce observed hierarchies in quark mass matrices, following initial assumptions are also applied:

- The axial charges $Z(Q_L^i) = -Z(Q_R^i)$ has been taken throughout, which will break the discrete $Left - Right$ symmetry.

- A charged bi-doublet H is considered. With this, H and $\tilde{H}$ can be distinguished, such that top-bottom splitting can be explained by $\tan\beta \gg 1$. In addition, some terms in the scalar potential will be forbidden, too. So the new $U(1)_{family}$ charges is normalized by

$$
Z(H) = -Z(\tilde{H}) = +2 \tag{2.5.17}
$$

- With this, the charges of the third family are fixed (up to a global sign) as

$$
Z(Q_L^{\rm III}) = +1\,, \qquad Z(Q_R^{\rm III}) = -1\,. \tag{2.5.18}
$$

  and

$$
y_{33}\, \bar{Q}_L^{\rm III}\, H\, Q_R^{\rm III} + \text{h.c.} \tag{2.5.19}
$$

  is an allowed dim-4 Yukawa term, while $\bar{Q}_L^{\rm III}\, \tilde{H}\, Q_R^{\rm III}$ are forbidden.

- In order not to have dim-4 Yukawa terms for the remaining inter-generational combinations, one requires that $Z(Q^{\rm I,II}) \pm Z(Q^{\rm III}) \neq \pm Z(H)$ as the result of which one gets $Z(Q^{\rm I,II}) \neq \pm 1, \pm 3$.

- This leaves e.g.

$$Z(Q_L^{\rm II}) = 0\,, \qquad Z(Q_R^{\rm II}) = 0\,.$$

  as a candidate for a non-trivial charge assignment for the second generation. The charges for the doublet fields $\chi_{L,R}$ are constructed in such a way, that the dim-5 terms contribute to the charm and strange quark masses. This implies

$$Z(\chi_L) = 0\,, \qquad Z(\chi_R) = 0\,,$$

  and the dim-5 term

$$y_{22}\,\bar{Q}_L^{\rm II}\chi_L\chi_R^\dagger Q_R^{\rm II} + \tilde{y}_{22}\,\bar{Q}_L^{\rm II}\chi_L^c\chi_R^T Q_R^{\rm II} + \text{h.c.} \tag{2.5.20}$$

  is allowed.

- In order that the remaining inter-generational combinations do not contribute at dim-4 or dim-5 order, one has to exclude $Z(Q^{\rm I}) \neq \pm 0,1,2,3$, which follows from the constraints $Z(Q^{\rm I}) \pm Z(Q^{\rm II,III}) \neq \pm Z(H)$ and $Z(Q^{\rm I}) \pm Z(Q^{\rm II,III}) \neq \pm Z(\chi_L\chi_R^\dagger) = 0$.

- This leaves

$$Z(Q_L^{\rm I}) = -4\,, \qquad Z(Q_R^{\rm I}) = +4\,,$$

  as the simplest possible charge assignment for the first generation.

- To fix the charge assignment for the triplet fields Δ_L,Δ_R possible dim-6 contributions to the up- and down masses can be considered now and the simplest choice is

$$Z(\Delta_L) = -3\,, \qquad Z(\Delta_R) = +3\,,$$

  which allows for

$$y_{11}\,(\bar{Q}_L^{\rm I}\,\tau^a\,\tilde{H}\,\tau^b\,Q_R^{\rm I})\,\Delta_L^a\,(\Delta_R^\dagger)^b + \text{h.c.} \tag{2.5.21}$$

- One can then further induce generation-mixing terms on dim-6 level via the operators

$$y_{12}\,(\bar{Q}_L^{\rm I}\,\tau^a\,H\,\tau^b\,Q_R^{\rm II})\,\Delta_L^a\,(\Delta_R^\dagger)^b + y_{21}\,(\bar{Q}_L^{\rm II}\,\tau^a\,H\,\tau^b\,Q_R^{\rm I})\,\Delta_L^a\,(\Delta_R^\dagger)^b \\ + \text{h.c.} + (L \leftrightarrow R) \tag{2.5.22}$$

  and also

$$y_{13}\,(\bar{Q}_L^{\rm I}\,\chi_L\,\chi_R^T\,\tau^a\,Q_R^{\rm III})(\Delta_R^\dagger)^a + y_{31}\,(\bar{Q}_L^{\rm III}\,\chi_L\,\chi_R^T\,\tau^a\,Q_R^{\rm I})(\Delta_R^\dagger)^a \\ + \text{h.c.} + (L \leftrightarrow R) \tag{2.5.23}$$

  It is to be noticed here that there are no off-diagonal mass matrix elements between the second and third generation for this particular case. Similarly contributions from dim-7 and dim-8 can also be considered with keeping above constraints in mind.

With the constraints given above one can then be able to assign different values to $U(1)_{family}$ charges. The allowed values are given in Table(2.2) It is to be kept in mind that examples are shown for charges less or equal 4 and also cases where charges trivially differ by relative signs are not listed.

	$Z_{Q^{\rm I}_L} = -Z_{Q^{\rm I}_R}$	$Z_{Q^{\rm II}_L} = -Z_{Q^{\rm II}_R}$	$Z_{\chi_L} = -Z_{\chi_R}$	$Z_{\Delta_L} = -Z_{\Delta_R}$
Case I	4	0	0	1
Case II	-4	0	0	-3
Case III	$+3$	0	0	$+1$
Case IV	$+3$	0	0	$+3$
Case V	0	-4	-4	-3
Case VI	0	4	-4	$+1$
Case VII	-2	$+3$	-3	$+1$
Case VIII	0	3	-3	$+3$
Case IX	0	$+3$	-3	$+1$
Case X	$+2$	$+3$	-3	$+3$

Table 2.2: Possible charge assignments for quark and scalar fields, and its classification. The charges of the Higgs bi-doublet and the third generation are fixed as $Z_H = +2$ and $Z_{Q^{\rm III}_L} = -Z_{Q^{\rm III}_R} = +1$.

2.5.3 Mass matrices

After having assigned the above $U(1)_{family}$ charges, now one can write entries in Y_U and Y_D matrices. All four combinations of $U(1)_{family}$ given in Table(2.2) are discussed here one by one. Considering Case-I for the given combination of $U(1)_{family}$ the power counting for Y_U is given as

$$Y_U \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^2 & \epsilon^3 \\ 0 & \epsilon^3 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda} & y_{23}\dfrac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} \\ 0 & y_{32}\dfrac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} & y_{33}v\sin\beta \end{pmatrix} \quad (2.5.24)$$

where y_{22}, y_{23}, y_{32}, y_{33} are Yukawa coupling constants having order 1. The above matrices are written by taking the leading contribution for the corresponding entry, for example in Y_U for the element y_{22} the complete power counting includes $\epsilon^2 + \epsilon^6$ terms but as ϵ^6 can just be treated as correction so it can be ignored. Also the absolute values has been taken for the elements of the matrix. These matrices then can be diagonalized to get quark masses. Keeping in mind the same argument down-type matrix written as

$$
\begin{aligned}
Y_D &\sim \begin{pmatrix} 0 & \epsilon^4 & 0 \\ \epsilon^4 & \epsilon^2 & \epsilon^3 \\ 0 & \epsilon^3 & \epsilon^2 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & y_{12}\dfrac{v\sin\beta v_L v_R}{\Lambda} & 0 \\ y_{21}\dfrac{v\sin\beta v_L v_R}{\Lambda^2} & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda^2} & y_{23}\dfrac{v_R\lambda_R\lambda_L}{\Lambda^2} \\ 0 & y_{32}\dfrac{v_R\lambda_R\lambda_L}{\Lambda^2} & y_{33}v\cos\beta \end{pmatrix}
\end{aligned} \tag{2.5.25}
$$

this down type matrix can the further be simplified by choosing $v_L \to 0$ as suggested in (add mahopatra reference). This will make the matrix simple and easy to diagonalize. Diagonalization of Y_U gives masses of the up quarks as $m_u \sim 0, m_c \sim \epsilon^2$ and $m_t \sim 1$. Similarly from Y_D one can get $m_d \sim 0, m_s \sim \epsilon^2$ and $m_b \sim \epsilon^2$. For case-II one gets

$$
\begin{aligned}
Y_U &\sim \begin{pmatrix} 0 & 0 & \epsilon^5 \\ 0 & \epsilon^2 & \epsilon^5 \\ \epsilon^5 & \epsilon^5 & 1 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & 0 & y_{13}\dfrac{v_L\lambda_L\lambda_R}{\Lambda^2} \\ 0 & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda} & y_{23}\dfrac{v\cos\beta v_R\lambda_R^2}{\Lambda^3} \\ y_{31}\dfrac{v_L\lambda_L\lambda_R}{\Lambda^2} & y_{32}\dfrac{v\cos\beta\ v_R\lambda_R^2}{\Lambda^3} & y_{33}v\sin\beta \end{pmatrix}
\end{aligned} \tag{2.5.26}
$$

Y_U obtained is in triangular form it shows the hierarchy of quark masses clearly. Likewise down-type matrices can be written as

$$
\begin{aligned}
Y_D &\sim \begin{pmatrix} \epsilon^6 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & \epsilon^2 \end{pmatrix} \\
&\sim \begin{pmatrix} \dfrac{y_{11}v\cos\beta v_L v_R}{\Lambda^2} & y_{12}\dfrac{v\sin\beta v_L v_R}{\Lambda^2} & y_{13}\dfrac{v_R\lambda_L\lambda_R}{\Lambda^2} \\ y_{21}\dfrac{v\sin\beta v_L v_R}{\Lambda^2} & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda} & y_{23}\dfrac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} \\ y_{31}\dfrac{v_R\lambda_L\lambda_R}{\Lambda^2} & y_{32}\dfrac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} & y_{33}v\cos\beta \end{pmatrix}
\end{aligned} \tag{2.5.27}
$$

For Case-III one write

$$
\begin{aligned}
Y_U &\sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^2 & \epsilon^3 \\ 0 & \epsilon^3 & 1 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda} & y_{23}\dfrac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} \\ 0 & y_{32}v\sin\beta\dfrac{v_R\lambda_R^2}{\Lambda^3} & y_{33}v\sin\beta \end{pmatrix}
\end{aligned} \tag{2.5.28}
$$

Likewise down-type matrice can be written as

$$
\begin{aligned}
Y_D &\sim \begin{pmatrix} 0 & \epsilon^{11} & \epsilon^4 \\ \epsilon^{11} & \epsilon^2 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & 0 & y_{13}\frac{v\sin\beta v_L v_R}{\Lambda^2} \\ 0 & y_{22}\frac{\lambda_L\lambda_R}{\Lambda} & y_{23}\frac{v_R\lambda_R\lambda_L}{\Lambda^2} \\ y_{31}\frac{v\sin\beta v_L v_R}{\Lambda^2} & y_{32}\frac{v_R\lambda_R\lambda_L}{\Lambda^2} & y_{33}v\cos\beta \end{pmatrix}
\end{aligned}
\tag{2.5.29}
$$

For Case-IV one write

$$
\begin{aligned}
Y_U &\sim \begin{pmatrix} 0 & \epsilon^5 & 0 \\ \epsilon^5 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & y_{12}\frac{v_L\lambda_L\lambda_R}{\Lambda^2} & 0 \\ y_{21}\frac{v_L\lambda_L\lambda_R}{\Lambda^2} & y_{22}\frac{\lambda_L\lambda_R}{\Lambda^2} & 0 \\ 0 & 0 & y_{33}v\sin\beta \end{pmatrix}
\end{aligned}
\tag{2.5.30}
$$

Likewise down-type matrice can be written as

$$
\begin{aligned}
Y_D &\sim \begin{pmatrix} 0 & \epsilon^3 & \epsilon^6 \\ \epsilon^3 & \epsilon^2 & \epsilon^8 \\ \epsilon^6 & \epsilon^8 & \epsilon^2 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & y_{12}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} & y_{13}\frac{v\cos\beta v_L v_R}{\Lambda^2} \\ y_{21}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} & y_{22}\frac{\lambda_L\lambda_R}{\Lambda^2} & y_{23}\frac{(v\cos\beta)^2 v\sin\beta}{\Lambda^2} \\ y_{31}\frac{v\cos\beta v_L v_R}{\Lambda^2} & y_{32}\frac{(v\cos\beta)^2 v\sin\beta}{\Lambda^2} & y_{33}v\cos\beta \end{pmatrix}
\end{aligned}
\tag{2.5.31}
$$

For Case-V one write

$$
\begin{aligned}
Y_U &\sim \begin{pmatrix} 0 & 0 & \epsilon^7 \\ 0 & 0 & \epsilon^3 \\ \epsilon^7 & \epsilon^3 & 1 \end{pmatrix} \\
&\sim \begin{pmatrix} 0 & 0 & y_{13}\frac{v\cos\beta v_R\lambda_L\lambda_R}{\Lambda^3} \\ 0 & 0 & y_{23}\frac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} \\ y_{31}\frac{v\cos\beta v_R\lambda_L\lambda_R}{\Lambda^3} & y_{32}\frac{v\sin\beta v_R\lambda_R^2}{\Lambda^3} & y_{33}v\sin\beta \end{pmatrix}
\end{aligned}
\tag{2.5.32}
$$

Likewise down-type matrice can be written as

$$
Y_D \sim \begin{pmatrix} 0 & \epsilon^4 & \epsilon^5 \\ \epsilon^4 & \epsilon^2 & \epsilon^3 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{pmatrix}
$$

$$
\sim \begin{pmatrix} 0 & y_{12}\frac{v\sin\beta v_L v_R}{\Lambda^2} & y_{13}\frac{v\sin\beta v_R\lambda_R\lambda_L}{\Lambda^3} \\ y_{21}\frac{v\sin\beta v_L v_R}{\Lambda^2} & y_{22}\frac{\lambda_L\lambda_R}{\Lambda^2} & y_{23}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} \\ y_{31}\frac{v\sin\beta v_R\lambda_R\lambda_L}{\Lambda^3} & y_{32}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} & y_{33}v\cos\beta \end{pmatrix} \tag{2.5.33}
$$

For Case-VI one write

$$
Y_U \sim \begin{pmatrix} \epsilon^6 & 0 & \epsilon^5 \\ 0 & \epsilon^2 & 0 \\ \epsilon^5 & 0 & 1 \end{pmatrix}
$$

$$
\sim \begin{pmatrix} y_{11}\frac{v\cos\beta v_R v_L}{\Lambda^2} & 0 & y_{13}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} \\ 0 & y_{22}\frac{\lambda_L\lambda_R}{\Lambda} & 0 \\ y_{31}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} & 0 & y_{33}v\sin\beta \end{pmatrix} \tag{2.5.34}
$$

Likewise down-type matrice can be written as

$$
Y_D \sim \begin{pmatrix} \epsilon^6 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & 0 & 0 \\ \epsilon^3 & 0 & \epsilon^2 \end{pmatrix}
$$

$$
\sim \begin{pmatrix} y_{11}\frac{v\cos\beta v_L v_R}{\Lambda^2} & y_{12}\frac{v\sin\beta v_L v_R}{\Lambda^2} & y_{13}\frac{v_R\lambda_R\lambda_L}{\Lambda^2} \\ y_{21}\frac{v\sin\beta v_L v_R}{\Lambda^2} & 0 & 0 \\ y_{13}\frac{v_R\lambda_R\lambda_L}{\Lambda^2} & 0 & y_{33}v\cos\beta \end{pmatrix} \tag{2.5.35}
$$

For Case-VII one write

$$
Y_U \sim \begin{pmatrix} \epsilon^4 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^2 & 0 \\ \epsilon^3 & 0 & 1 \end{pmatrix}
$$

$$
\sim \begin{pmatrix} y_{11}\frac{v\sin\beta v_R v_L}{\Lambda^2} & y_{12}\frac{v_L\lambda_L\lambda_R}{\Lambda^2} & y_{13}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} \\ y_{21}\frac{v_L\lambda_L\lambda_R}{\Lambda^2} & y_{22}\frac{\lambda_L\lambda_R}{\Lambda} & 0 \\ y_{31}\frac{v_R\lambda_L\lambda_R}{\Lambda^2} & 0 & y_{33}v\sin\beta \end{pmatrix} \tag{2.5.36}
$$

Likewise down-type matrice can be written as

$$Y_D \sim \begin{pmatrix} 0 & \epsilon^3 & \epsilon^9 \\ \epsilon^3 & 0 & \epsilon^4 \\ \epsilon^9 & \epsilon^4 & \epsilon^2 \end{pmatrix}$$
$$\sim \begin{pmatrix} 0 & y_{12}\dfrac{\lambda_L\lambda_R v_R}{\Lambda^2} & y_{13}\dfrac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} \\ y_{21}\dfrac{\lambda_L\lambda_R v_R}{\Lambda^2} & 0 & y_{23}\dfrac{v\sin\beta v_L v_R}{\Lambda^2} \\ y_{31}\dfrac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} & y_{32}\dfrac{v\sin\beta v_L v_R}{\Lambda^2} & y_{33} v\cos\beta \end{pmatrix} \quad (2.5.37)$$

For Case-VIII one write

$$Y_U \sim \begin{pmatrix} 0 & \epsilon^5 & 0 \\ \epsilon^5 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\sim \begin{pmatrix} 0 & y_{12}\dfrac{v_L\lambda_L\lambda_R}{\Lambda^2} & 0 \\ y_{21}\dfrac{v_L\lambda_L\lambda_R}{\Lambda^2} & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda} & 0 \\ 0 & 0 & y_{33} v\sin\beta \end{pmatrix} \quad (2.5.38)$$

Likewise down-type matrice can be written as

$$Y_D \sim \begin{pmatrix} 0 & \epsilon^3 & \epsilon^9 \\ \epsilon^3 & 0 & \epsilon^6 \\ \epsilon^9 & \epsilon^6 & \epsilon^2 \end{pmatrix}$$
$$\sim \begin{pmatrix} 0 & y_{12}\dfrac{\lambda_L\lambda_R v_R}{\Lambda^2} & y_{13}\dfrac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} \\ y_{21}\dfrac{\lambda_L\lambda_R v_R}{\Lambda^2} & 0 & y_{23}\dfrac{v\cos\beta v_L v_R}{\Lambda^2} \\ y_{31}\dfrac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} & y_{32}\dfrac{v\cos\beta v_L v_R}{\Lambda^2} & y_{33} v\cos\beta \end{pmatrix} \quad (2.5.39)$$

For Case-IX one write

$$Y_U \sim \begin{pmatrix} \epsilon^6 & 0 & \epsilon^5 \\ 0 & \epsilon^2 & 0 \\ \epsilon^5 & 0 & 1 \end{pmatrix}$$
$$\sim \begin{pmatrix} y_{11}\dfrac{v\cos\beta v_L v_R}{\Lambda^2} & 0 & y_{13}\dfrac{v_L\lambda_L\lambda_R}{\Lambda^2} \\ 0 & y_{22}\dfrac{\lambda_L\lambda_R}{\Lambda} & 0 \\ y_{31}\dfrac{v_L\lambda_L\lambda_R}{\Lambda^2} & 0 & y_{33} v\sin\beta \end{pmatrix} \quad (2.5.40)$$

Likewise down-type matrice can be written as

$$
\begin{aligned}
Y_D \sim & \begin{pmatrix} \epsilon^6 & \epsilon^9 & \epsilon^3 \\ \epsilon^9 & 0 & \epsilon^4 \\ \epsilon^3 & \epsilon^4 & \epsilon^2 \end{pmatrix} \\
\sim & \begin{pmatrix} y_{11}\frac{\cos\beta v_L v_R}{\Lambda^2} & y_{12}\frac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} & y_{13}\frac{\lambda_L\lambda_R v_R}{\Lambda^2} \\ y_{21}\frac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} & 0 & y_{23}\frac{v\sin\beta v_L v_R}{\Lambda^2} \\ y_{31}\frac{\lambda_L\lambda_R v_R}{\Lambda^2} & y_{32}\frac{v\sin\beta v_L v_R}{\Lambda^2} & y_{33} v\cos\beta \end{pmatrix}
\end{aligned} \tag{2.5.41}
$$

For Case-X one write

$$
\begin{aligned}
Y_U \sim & \begin{pmatrix} 0 & 0 & \epsilon^5 \\ 0 & \epsilon^2 & 0 \\ \epsilon^5 & 0 & 1 \end{pmatrix} \\
\sim & \begin{pmatrix} 0 & 0 & y_{13}\frac{v_L\lambda_L\lambda_R}{\Lambda^2} \\ 0 & y_{22}\frac{\lambda_L\lambda_R}{\Lambda} & 0 \\ y_{31}\frac{v_L\lambda_L\lambda_R}{\Lambda^2} & 0 & y_{33} v\sin\beta \end{pmatrix}
\end{aligned} \tag{2.5.42}
$$

Likewise down-type matrice can be written as

$$
\begin{aligned}
Y_D \sim & \begin{pmatrix} \epsilon^6 & \epsilon^9 & \epsilon^3 \\ \epsilon^9 & 0 & \epsilon^6 \\ \epsilon^3 & \epsilon^6 & \epsilon^2 \end{pmatrix} \\
\sim & \begin{pmatrix} y_{11}\frac{\cos\beta v_L v_R}{\Lambda^2} & y_{12}\frac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} & y_{13}\frac{\lambda_L\lambda_R v_R}{\Lambda^2} \\ y_{21}\frac{(v\cos\beta)^2 v\sin\beta v_L}{\Lambda^2} & 0 & y_{23}\frac{v\cos\beta v_L v_R}{\Lambda^2} \\ y_{31}\frac{\lambda_L\lambda_R v_R}{\Lambda^2} & y_{32}\frac{v\cos\beta v_L v_R}{\Lambda^2} & y_{33} v\cos\beta \end{pmatrix}
\end{aligned} \tag{2.5.43}
$$

One of the significance of the $U(1)_{family}$ symmetry is the possibility of getting zeros in quark mass matrices. The presence of which opens a possibility of exact relations among various, otherwise independent, parameters of the quark sector. As symmetries applied in the theories beyond SM can reduce the number of free parameters of the Yukawa coupling matrices, giving relationships between the CKM matrix elements and the quark masses. The first relationship so obtained in the gauge theory was the very successful prediction for the Cabibbo angle given as $|V_{us}| = \sqrt{\frac{m_d}{m_s}}$ [30] [31], where $|V_{us}| = 0.221 \pm 0.002$ and $\sqrt{\frac{m_d}{m_s}} = 0.226 \pm 0.009$[32]. Much interest has also centered around the relation $|V_{cb}| = \sqrt{\frac{m_c}{m_t}}$ obtained by Harvey, Ramond and Reiss [33] working with the form for the Yukawa matrices written by Georgi and jarlskog [34]. If this relation is valid at the weak scale the top quark would be

predicted to be too heavy. The relations like $\frac{V_{ub}}{V_{cb}} = \frac{m_u}{m_c}$ and $\frac{V_{td}}{V_{ts}} = \frac{m_d}{m_s}$ are significant successes of some specific models for quark masses [35][36][37].
The masses obtained after diagonalization of the matrices then can be used to get the mixing angles and hence can be used to get CKM matrix elements.

2.6 Flavor Effects

LRSMs also gives basis for the study of right handed charged currents . The effect of right-handed W- coupling on the extraction of $|V_{ub}|$ and $|V_{cb}|$ is studied extensively in literature. The impact of right-handed currents can be studied by denoting CKM element extracted from data with SM formula by $V_{q,b}$, where $q = u$ or $q = c$. If the matrix element of an exclusive process is proportional to the vector current, V_{qb}^L and V_{qb}^R enter with the same sign and the "true" value of V_{qb}^L in the presence of V_{qb}^R is given by

$$V_{qb}^L = V_{qb} - V_{qb}^R \tag{2.6.1}$$

For the processes proportional to the axial-vector current V_{qb}^R enters with the opposite sign as V_{qb}^L so that

$$V_{qb}^L = V_{qb} + V_{qb}^R \tag{2.6.2}$$

The relative uncertainties in the exclusive decays $\bar{B} \to D^* l\bar{\nu}$ and $\bar{B} \to Dl\bar{\nu}$ and in inclusive $B \to X_c l\nu$ analysis are much smaller than in $b \to u$ decays. The decay $\bar{B} \to Dl\bar{\nu}$ only involves the vector current so only Eq.(2.6.1) is applied. However $\bar{B} \to D^* l\bar{\nu}$ receives contributions from both vector and axial vector current but the contribution from the vector current is suppressed in the kinematical end point region used for the extraction of $|V_{cb}|$. Therefore Eq.(2.6.2) applies to $\bar{B} \to D^* l\bar{\nu}$.

Although none of the new vector boson is observed so far the numerical bound derived from the lack of signal depends on some assumptions, in particular concerning the strength of the relevant effective gauge coupling and whether the decay $W_R^+ \to l^+\nu_R$ can occur, left - and right- handed neutrinos can possess vastly different mass which turns out to be one of the most attractive features of Left-Right models. Also with these models one can search for the right handed currents in $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ through a detailed analysis of the generalized Michel parameters.

As stated earlier one interesting point in the discussion of LRSMs is the possibility of explaining the observed CP violation. Standard Model is able to explain it because of three families of particles with the help of which, it is possible to absorb all the complex phases arising from the Yukawa sector of the Lagrangian except for one. This remaining phase appears in the CKM matrix and thus can explain the CP violation in the kaon system [38]–[44] observed experimentally.

Presence of $SU(2)_R$ doublets, V_{CKM}^R is also obtained along with the usual Standard Model CKM matrix V_{CKM}^L. One can however impose a global CP symmetry on the complete Lagrangian, in order to avoid explicit complex phases in the Yukawa couplings, and obtain them spontaneously through the vacuum expectation values of Higgs bi-doublet arising from the symmetry breaking mechanism [45]–[56]. This is the case known as pseudo left-right symmetry in which CP violation arises through the vevs of scalar Higgs particles. It should be kept in mind that a model with right handed currents in general contains a sizable number of physical CP violating phases emanating from the mass matrices: for N families they number $[(N-1)(N-2)+N(N+1)]/2$ i.e. 1,3,6 for $N=1$,2,3 respectively. Taking into account the phases in vev's of scalars as one example one can write

$$\begin{aligned}\langle H\rangle &= \begin{pmatrix} k_1 e^{i\alpha_1} & 0 \\ 0 & k_2 e^{i\alpha_2}, \end{pmatrix} \\ \langle \Delta_L\rangle &= \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0, \end{pmatrix} \\ \langle \Delta_R\rangle &= \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta_R} & 0, \end{pmatrix}\end{aligned} \tag{2.6.3}$$

Under Unitarity transformations of the fermionic fields, the scalar fields transforms according to the relations:

$$\begin{aligned}H &= U_L H U_R^\dagger \\ \Delta_L &= U_L \Delta_L U_L^\dagger \\ \Delta_R &= U_R \Delta_R U_R^\dagger\end{aligned} \tag{2.6.4}$$

Some of the phases can be absorbed by redefining the scalar fields with only two phases left obtained spontaneously.

$$\begin{aligned}U_L &= \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{i\gamma_L} \end{pmatrix} \\ U_R &= \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{i\gamma_r} \end{pmatrix} \\ \gamma_L &= \frac{\theta_L}{2}, \\ \gamma_R &= \gamma_L - \alpha_2\end{aligned} \tag{2.6.5}$$

As a result two spontaneous CP phases appear,α and θ, which may be allocated in the CKM matrix and in the analogous matrix for the lepton sector respectively making it interesting as it opens the possibility of having CP violation in the lepton sector too, which is not possible in the SM. However, due to the presence of additional scalar fields, Flavor Changing Neutral Currents (FCNC) appear in the model involving the neutral scalar bosons [45]–[48]. The experimental constraints tell us that if the FCNC exist, they must be suppressed enough so that the experiments will not be sensitive to them [23]. One possible way to avoid the FCNC in the LRSMs, without doing any fine-tuning on the coupling constants, is to have really heavy masses for the scalar bosons mediating the FCNC.

Chapter 3

Right-Handed Currents in $B \to D(D^*)l\nu$ decays

The CKM sector of Standard Model can be tested by the precise measurements of $|V_{ub}|$ and $|V_{cb}|$. One of the most accurate methods for the extraction of these values is the study of exclusive semileptonic B-meson decays. The theoretical framework used to study such decays is based on the fact that the mass m_b of b-quark is very large compared to the scale of Λ_{QCD}, which determines the low-energy hadronic physics, as a result it is a good approximation to take the $m_b \to \infty$ limit. In this limit QCD has a spin-flavor heavy quark symmetry, which has important implications for the properties of hadrons containing a single heavy quark. The interactions of such heavy quarks with light quarks and gluons can then be described by an effective theory which is invariant under the change of the flavor and spin of the heavy quark[59]. The existence of an exact symmetry limit of a theory increases the prospects for the precise determination of the elements of the quark mixing matrix.

3.1 Heavy-quark effective theory

Effective theories are one of the important tools in theoretical physics. These effective theories based on the fact that study of high energy physics effects of very heavy particle become irrelevant at low energies so the heavy degrees of freedom can be removed without altering the basic physics. The process of removing the degrees of freedom of a heavy particle involves the identification of the heavy particle fields and integrating them out in the generating functional of the Green functions of the theory. As the action of the full theory is usually a local one, integrating the particle field out results in a non local effective action. This effective action is rewritten as an infinite series of local terms in an operator product expansion [60],[61]. This expansion in $1/m_Q$ of the heavy quark mass then separates the short- and long- distance physics. The long-distance physics corresponds to interactions at low energies and is the same in the full and effective theory, but short-distance effects arising from quantum corrections involving large virtual momenta are not reproduced in the effective theory. They have to be added in a perturbative way using renormalization group techniques. This procedure is called matching the effective theory to the full theory. It leads to renormalization of the coefficient of the local operators in the effective Lagrangian.

3.1.1 Effective Lagrangian

The starting point in based on the observation that heavy quark bound inside a hadron moves with the hadron's velocity v^μ and it is almost on-shell. This heavy quark momentum can be given as [62]

$$p_Q^\mu = m_Q v^\mu + k^\mu, \tag{3.1.1}$$

where v^μ is the hadron four-velocity satisfying the relation $v^2 = 1$ and k^μ is the residual momentum of order Λ_{QCD} and determines the amount by which the quark is off shell because of its interactions. The usual Dirac quark propagator in heavy quark limit simplifies to

$$i\frac{\not p + m_Q}{p^2 - m_Q^2 + i\epsilon} = i\frac{m_Q \not v + m_Q + \not k}{2m_Q v\cdot k + k^2 + i\epsilon} \to i\frac{1+\not v}{2v\cdot k + i\epsilon} \tag{3.1.2}$$

The propagator contains a velocity-dependent projection operator

$$\frac{1+\not v}{2} \tag{3.1.3}$$

It is convenient to formulate the effective Lagrangian directly in terms of velocity-dependent fields $Q(x)$, which are related to the original quark fields at tree level. Hence the heavy quark field can then be written as

$$Q(x) \equiv (P_+ + P_-)\, Q(x) \equiv \exp\left(-im_Q v.x\right)\left(h_v^{(Q)}(x) + H_v^{(Q)}(x)\right), \tag{3.1.4}$$

where the large and small component fields $h_v^{(Q)}$ and $H_v^{(Q)}$ are introduced as

$$h_v^{(Q)}(x) = \exp\left(im_Q v.x\right) P_+ Q(x), \qquad H_v^{(Q)}(x) = \exp\left(im_Q v.x\right) P_- Q(x), \tag{3.1.5}$$

where the projectors are given as $P_\pm = \frac{(1\pm\not v)}{2}$. It is to be noted that because of the projection operators, the new fields satisfies the relations $\not v h_v^{(Q)} = h_v^{(Q)}$ and $\not v H_v^{(Q)} = -H_v^{(Q)}$. In the hadron rest frame, i.e. $v^\mu = \left(1,\vec{0}\right), P_\pm = (1\pm\gamma_0)/2$; thus, $h_v^{(Q)}(x)$ and $H_v^{(Q)}(x)$ corresponds to the upper and lower components of $Q(x)$, receptively. The field $h_v^{(Q)}(x)$ annihilates a heavy quark with velocity v^μ, while $H_v^{(Q)}(x)$ creates a heavy anti-quark with the same velocity.

As energy scale of interest is $k \ll m_Q$, in which heavy antiquary cannot be produced, so $H_v^{(Q)}(x)$ should be integrated out . The field redefinition Eq.(3.1.4) is only adequate for describing a heavy quark. If study of heavy anti quark is under consideration then one should use

$$Q(x) \equiv (P_- + P_+)\, Q(x) \equiv e^{im_Q v.x}\left(h_v^{-(q)}(x) + H_v^{-(Q)}(x)\right). \tag{3.1.6}$$

The anti quark formalism is identical to the quark one, with the replacements $v^\mu \to -v^\mu$ and $h_v^{(Q)}(x) \to h_v^{-(Q)}(x)$.

With the redefinition Eq.(3.1.4), the heavy-quark Lagrangian takes the form

$$\begin{aligned}\mathcal{L}^{(Q)}_{QCD} &= \left(\bar{h}^{(Q)}_v + \bar{H}^{(Q)}_v\right)\left[i\not{D} - 2m_Q P_-\right]\left(h^{(Q)}_v + H^{(Q)}_v\right) \\ &= \bar{h}^{(Q)}_v i\,(v.D)\,h^{(Q)}_v - \bar{H}^{(Q)}_v\,(iv.D + 2m_Q)\,H^{(Q)}_v \\ &\quad + \bar{h}^{(Q)}_v i\,(D_\perp)\,H^{(Q)}_v + \bar{H}^{(Q)}_v i\left(\not{D}_\perp\right)h^{(Q)}_v, \end{aligned} \tag{3.1.7}$$

where $D^\mu_\perp \equiv D^\mu - v^\mu\,(v.D)$ is the component of the Dirac operator orthogonal to velocity, i.e. $v.D_\perp = 0$ and the relations $P_\pm\gamma^\mu P_\pm = P_\pm v^\mu P_\pm$ and $P_\mp\not{D}P_\pm = P_\mp\not{D}_\perp P_\pm$ has been used to obtain the result. In the hadron rest frame, $D^\mu_\perp = \left(0,\vec{D}\right)$ contains just the space components of covariant derivative.

From Eq.(3.1.7), it is apparent that field $h^{(Q)}_v$ describes a mass less degree of freedom, while $H^{(Q)}_v$ corresponds to fluctuations with an energy at least twice the heavy mass. These are the heavy degrees of freedom that will be eliminated in the construction of the effective theory. The third and fourth term in Eq.(3.1.7), which mix the two field, describes quark-anti quark creation annihilation. A virtual heavy quark propagating forward in time can turn into a virtual anti quark propagating backward in time and then turns back into quark. Since there is no energy to produce on-shell quark-anti quark pairs, the virtual fluctuation into the intermediate $h^{(Q)}_v h^{(Q)}_v \bar{H}^{(Q)}_v$ state can propagate over a short distance $\triangle x \sim 1/m_Q$. On hadronic scale this process looks like a local interaction of the form Fig.(3.1)

$$h^{\bar{(Q)}}{}_v i\not{D}_\perp\,(1/2m_Q)\,i\not{D}_\perp h^{(Q)}_v, \tag{3.1.8}$$

where the propagator for $H^{(Q)}_v$ is simply replaced by $i/2m_Q$.

At the classical level, one can eliminate the heavy degrees of freedom represented as $H^{(Q)}_v$ by using equation of motion $\left(i\not{D} - m_Q\right)Q = 0$, which in terms of the $h^{(Q)}_v$ and $H^{(Q)}_v$ fields takes the form

$$i\not{D}h^{(Q)}_v + \left(-i\not{D}_\perp - 2m_Q\right)H^{(Q)}_v = 0 \tag{3.1.9}$$

Multiplying it by $P_\pm$, this equation gets projected into two different pieces:

$$iv.Dh^{(Q)}_v = -i\not{D}_\perp H^{(Q)}_v; \qquad (iv.D + 2m_Q)\,H^{(Q)}_v = i\not{D}_\perp h^{(Q)}_v. \tag{3.1.10}$$

The second shows explicitly that $H^{(Q)}_v \sim \mathcal{O}\,(1/m_Q)$:

$$H^{(Q)}_v = \frac{1}{iv.D + 2M_Q - i\epsilon}\,i\not{D}_\perp h^{(Q)}_v. \tag{3.1.11}$$

Inserting Eq.(3.1.11) back into Eq.(3.1.7), one gets the effective Lagrangian.

$$\mathcal{L}_{eff} = \bar{h}^{(Q)}_v i\,(v.D)\,h^{(Q)}_v + \bar{h}^{(Q)}_v i\not{D}_\perp\frac{1}{iv.D + 2m_Q - i\epsilon}\,i\not{D}_\perp h^{(Q)}_v. \tag{3.1.12}$$

The second term corresponds to the virtual quark-anti quark fluctuations of $\mathcal{O}(1/m_Q)$ depicted in Fig (3.1)

This Lagrangian can also be obtained in more elegant way, manipulating generating functional for QCD Green functions containing heavy quark fields. To this end,

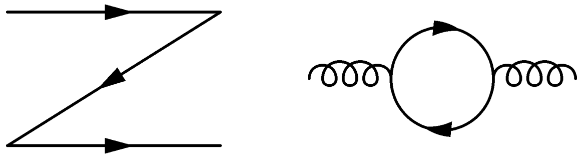


Figure 3.1: Virtual fluctuations involving pair creation of heavy quarks. Time flows to the right.

one starts from the field redefinition Eq.(3.1.6) and couples the large-component fields $h_v^{(Q)}$ to external sources ρ_v. Green functions with an arbitrary number of $h_v^{(Q)}$ fields can be constructed by taking derivatives with respect to ρ_v. No sources are needed for the heavy degrees of freedom represented by $H_v^{(Q)}$. The functional integration over the $H_v^{(Q)}$ field is Gaussian and can be performed explicitly, leading to the effective action

$$S_{eff} = \int \mathrm{d}^4x \mathcal{L}_{eff} - i\ln\triangle, \tag{3.1.13}$$

with $\mathcal{L}_{eff}$ as given in Eq.(3.1.7). The appearance of the logarithm of the determinant

$$\triangle = \det\left(iv\cdot D + 2m_Q - i\epsilon\right)^{1/2} = \exp\left\{\frac{1}{2}\mathrm{tr}[\log(iv\cdot D + 2m_Q - i\epsilon)]\right\}, \tag{3.1.14}$$

which is a quantum effect not present in the classical derivation presented above. However, by choosing the axial Gauge $v\cdot G = 0$, one can easily see that Eq.(3.1.14) is just an irrelevant constant [63][64].

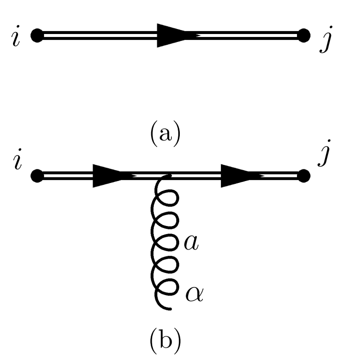
3.1.2 $1/M_Q$ Expansion

Because of phase factor in Eq.(3.1.6) the x-dependence of the effective field $h_v^{(Q)}$ is rather weak i.e., the Fourier transform of $h_v^{(Q)}$ contains only the small residual momenta k^μ. Derivatives acting on $h_v^{(Q)}$ produce powers of the momentum k^μ, which is much smaller than m_Q. Therefore, the non-local HQET Lagrangian Eq.(3.1.12) can be expanded in powers of iD/m_Q:

$$\mathcal{L}_{HQET} = \bar{h}_v^{(Q)} iv\cdot D h_v^{(Q)} + \frac{1}{2m_Q}\sum_{n=0}^{\infty}\bar{h}_v^{(Q)} i\not{D}_\perp\left(-\frac{v\cdot D}{2m_Q}\right)^n i\not{D}_\perp h_v^{(Q)}. \tag{3.1.15}$$

Taking into account that $h_v^{(Q)}$ contains a P_+ projection operator, and using the identity

$$\begin{aligned} P_+ i\not{D}_\perp i\not{D}_\perp P_+ &= P_+\{\left(i\not{D}_\perp\right)^2 + \frac{1}{2}\left[i\not{D} i\not{D}\right]\}P_+ \\ &= P_+\{\left(i\not{D}_\perp\right)^2 + \frac{g}{2}\sigma_{\alpha\beta}G^{\alpha\beta}\}P_+ \end{aligned} \tag{3.1.16}$$



$$= \frac{i}{v \cdot k} \frac{1 + \not{v}}{2} \delta_{ji}$$

$$= i g_s v^\alpha (T_a)_{ji}$$

Figure 3.2: Feynman rules of the HQET (i,j and a are color indices). A heavy quark with velocity v is represented by the double line.

where $i\left[D^\alpha, D^\beta\right] = g_s G^{\alpha\beta}$ and $G^{\alpha\beta} \equiv \frac{\lambda^a}{2} G_a^{\alpha\beta}$ the gluon field strength tensor, one finds [69][70]

$$\begin{aligned}\mathcal{L}_{HQET} &= \bar{h}_v^{(Q)} i\,(v.D)\, h_v^{(Q)} + \frac{1}{2m_Q} \bar{h}_v^{(Q)} \left(i\not{D}\right)^2 h_v^{(Q)} \\ &\quad + \frac{g}{4m_Q} \bar{h}_v^{(Q)} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(Q)} + \mathcal{O}(1/m_Q^2). \end{aligned} \tag{3.1.17}$$

In the limit $m_Q \to \infty$, only first term remains:

$$\mathcal{L}_\infty = \bar{h}_v^{(Q)} i v \cdot D h_v^{(Q)}. \tag{3.1.18}$$

This is the effective Lagrangian of HQET. It gives rise to the Feynman rules shown in Fig(3.2) The first of the $1/m_Q$ order terms can be identified in the rest frame as

$$O_{kin} = \frac{1}{2m_Q} \bar{h}_v \left(iD_\perp\right)^2 h_v \to -\frac{1}{2m_Q} \bar{h}_v \left(iD\right)^2 h_v \tag{3.1.19}$$

is only the gauge covariant extension of kinetic energy which arises due to the off-shell residual motion of heavy quark. The second term describes the interaction of the heavy-quark spin with the gluon field

$$O_{mag} = \frac{g}{4m_Q} \bar{h}_v^{(Q)} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(Q)} \to -\frac{g}{m_Q} \bar{h}_v^{(Q)} S.B_c h_v^{(Q)}. \tag{3.1.20}$$

where S is the spin operator given as

$$\vec{S} \equiv \frac{1}{2} \gamma_5 \gamma^0 \vec{\gamma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \tag{3.1.21}$$

satisfies the relations

$$\left[S^i, S^j\right] = i\epsilon^{ijk} S^k, \qquad \left[\not{v}, S^i\right] = 0 \tag{3.1.22}$$

and $B_c^i = -\frac{1}{2}\epsilon^{ijk} G^{jk}$ are the components of the color magnetic gluon field. This chromomagnetic hyperfine interaction is a relativistic effect, which scales like $1/m_Q$

Using expression in Eq(3.1.11) for $H_v^{(Q)}$, obtained from the equation of motion,one can also derive a $1/m_Q$ expansion for the full heavy-quark field $Q(x)$

$$\begin{aligned} Q(x) &= \exp\left(-im_Q v.x\right)\left(1+\frac{1}{(iv.D+2m_Q-i\epsilon)}i\not{D}_{\perp}\right)h_v(x) \\ & \exp\left(-im_Q v.x\right)\left(1+i\not{D}_{\perp}/2m_Q+\cdots\right)h_v(x). \end{aligned} \tag{3.1.23}$$

With the help of the above relation any operator in HQET that contains one or more heavy quarks can be constructed at tree level. Vector current $V^\mu = \bar{q}\gamma^\mu Q$ composed of heavy and light quark can be represented as

$$V^\mu = \exp\left(-im_Q v.x\right)\bar{q}(x)\gamma^\mu\left(1+i\not{D}_{\perp}/2m_Q+\cdots\right)h_v(x) \tag{3.1.24}$$

$$\equiv \exp^{-im_Q v.x} V^\mu(x)_{\text{HQET}} \tag{3.1.25}$$

effective theory makes the m_Q-dependence of form factors of the matrix elements which are parametrized by the hadronic form factors which will be discussed letter.

3.1.3 Heavy Quark symmetries

Heavy quark limit gives two additional symmetries which are absent in the full QCD Lagrangian and they restrict the long-distance contributions in model independent way. One of them is the heavy-flavor symmetry. The interaction of quark with gluon is independent of the flavor (flavor dependence in full QCD is only because of different quark masses). At leading order to $1/m_Q$, the HQET Lagrangian is mass-independent as a result a symmetry among the quarks moving with same velocity appears. For N_h heavy quarks moving at the same velocity, Eq.(3.1.18) can be extended by writing

$$\mathcal{L}_\infty = \sum_{i=1}^{N_h} \bar{h}_v^{i(Q)} iv\cdot D h_v^{i(Q)} \tag{3.1.26}$$

For the case of two flavors b and c, the Lagrangian can be written as[58]

$$\mathcal{L}_{heavy} = \bar{b}_v\left(v.D\right)b_v + \bar{c}_v\left(v.D\right)c_v \tag{3.1.27}$$

where b_v and c_v are field operator h_v for the b and c quarks, respectively. It is obvious that the above Lagrangian is invariant under $SU(2)_{HF}$ rotations.

$$\begin{pmatrix} b_v \\ c_v \end{pmatrix} \to U_v \begin{pmatrix} b_v \\ c_v \end{pmatrix}, \qquad U \in SU(2)_{HF}. \tag{3.1.28}$$

It has to kept in mind that this symmetry relates only those quarks which are moving with the same velocity.

The second symmetry is heavy-spin symmetry arising because of the phase factor in (3.1.4). As is clear from the Lagrangian in heavy-mass limit, both spin degrees of freedom of the heavy quark couple in the same way to the gluon, one may write the leading order Lagrangian as

$$\mathcal{L} = \bar{h}_v^{+s}\left(ivD\right)h_v^{+s} + \bar{h}_v^{-s}\left(ivD\right)h_v^{-s}, \tag{3.1.29}$$

where $h_v^{\pm s}$ are the projectors of the heavy-quark field on a definite spin direction s,

$$h_v^{\pm s} = \frac{1}{2}\left(1 \pm \gamma_5 \not{s}\right) h_v, \qquad s \cdot v = 0, \qquad = -1. \tag{3.1.30}$$

The Lagrangian has a symmetry under the rotations of the heavy-quark spin and hence all heavy hadron states moving with the velocity v fall into spin-symmetry doublets as $m_Q \to \infty$. In Hilbert space, this symmetry is generated by operators $S_v(\epsilon)$ as

$$[h_v, S_v(\epsilon)] = i\not{\epsilon}\not{v}\gamma_5 h_v, \tag{3.1.31}$$

where ϵ, with $\epsilon^2 = -1$, is the rotation axis. The simplest spin-symmetry doublet in the mesonic case consists of the pseudoscalar meson $M(v)$ and the corresponding vector meson $M^*(v,\epsilon)$, since a spin rotation yields

$$\exp\left(iS_v(\epsilon)\frac{\pi}{2}\right)|M(v)\rangle = (-i)|M^*(v,\epsilon)\rangle, \tag{3.1.32}$$

where an arbitrary phase $(-i)$ has been chosen.

The spin symmetry relation between the pseudoscalar and the vector meson can be implemented by using the representation matrices for these states

$$M(v) = \frac{1}{2}\sqrt{m_M}\gamma_5\left(\not{v} - 1\right) \qquad \text{for the pseudoscalar meson,} \tag{3.1.33}$$

$$M^*(v,\epsilon) = \frac{1}{2}\sqrt{m_M}\not{\epsilon}\left(\not{v} - 1\right) \qquad \text{for the vector meson} \tag{3.1.34}$$

where the two indices of the matrices correspond to the indices of the heavy quark and the light anti-quark respectively. Use of these matrices allows to exploit the heavy-quark spin symmetry in a simple way. If $\mathcal{M}(v)$ denotes either $M(v)$ or $M^*(v,\epsilon)$ and if $|\mathcal{M}(v)\rangle$ is the corresponding state, heavy-heavy transition current can be given as

$$\langle\mathcal{M}(v')|\bar{h}_{v'}\Gamma h_v|\mathcal{M}(v)\rangle = \xi(v.v')\mathrm{Tr}\left\{\bar{\mathcal{M}}(v')\Gamma\mathcal{M}(v)\right\}, \tag{3.1.35}$$

where Γ is some arbitrary combination of Dirac matrices. Eq.(3.1.35) is one of the most important results of heavy-quark symmetry in mesonic sector, since it relates matrix elements in heavy-heavy current to a single form factor, called Isgur-Wise function[88] [87] ξ. This Isgur-Wise function is just reduced matrix element which is universal for whole spin-flavor symmetry multiplet.

Since the current

$$j_\mu = \bar{h}_\nu\gamma_\mu h_\nu \tag{3.1.36}$$

generates the heavy- flavor symmetry, one gets a renormalization statement for the Isgur-Wise function

$$\xi(v.v' = 1) = 1 \tag{3.1.37}$$

as the generators of a symmetry have to have normalized matrix elements. In the heavy-mass limit, the spin-symmetry partners have to be degenerate and with $J = s \pm \frac{1}{2}$ and there splitting has to scale as $1/m_Q$,

$$m_M = m_Q + \bar{\Lambda} + \frac{1}{2m_Q}\left(\lambda_1 + d_M\lambda_2\right), \tag{3.1.38}$$

where $d_M = 3$ for the 0^- meson and $d_M = -1$ for 1^- meson. The parameters $\bar{\Lambda}$, λ_1 ,λ_2 correspond to matrix elements involving higher-order terms that appears in the effective-theory Lagrangian,

$$\begin{aligned}
\bar{\Lambda} &= \frac{\langle 0|qivD\gamma_5 h_v|H(v)\rangle}{\langle 0|q\gamma_5 h_v|H(v)\rangle}, && (3.1.39)\\
\lambda_1 &= \frac{\langle H(v)|\bar{h}_v(iD)^2 h_v|H(v)\rangle}{2M_M}, && (3.1.40)\\
\lambda_2 &= \frac{\langle H(v)|\bar{h}_v\sigma_{\mu\nu}iD^\mu iD^\nu h_v|H(v)\rangle}{2M_M}, && (3.1.41)
\end{aligned}$$

The normalization of states has been chosen to be $\langle H(v)|\bar{h}_v h_v|H(v)\rangle = 2m_M = 2(m_Q + \bar{\Lambda})$. Also the $\bar{\Lambda}$ is interpreted as the binding energy of the heavy meson in infinite mass limit, λ_1 as the expectation value of the kinetic energy of the heavy quark and λ_2 as the energy inside heavy meson due to chromomagnetic moment of the heavy quark. λ_1, λ_2 plays as important role as they parametrize the non-purtabative input needed in the sub leading order of the $1/m_Q$ expansion. The prediction in Eq.(3.1.38) from spin symmetry when checked against experimental data is quite a good approximation.

$$\begin{aligned}
M_{D^*} - M_D &= (142.12 \pm 0.07)\text{MeV},\\
\frac{M_{D^*} - M_D}{M_D} &\approx 8\%, && (3.1.42)\\
M_{B^*} - M_B &= (45.7 \pm 0.4)\text{MeV},\\
\frac{M_{B^*} - M_B}{M_B} &\approx 0.9\% && (3.1.43)
\end{aligned}$$

The infinite-mass limit works much better for the bottom, although the results are good for charm case. We expect these mass splittings to get corrections of the form $M_{P^*} - M_P \approx a/M_Q$; this gives the redefined prediction $M_{B^*}^2 - M_B^2 \approx M_{D^*}^2 - M_D^2$, which is a very good agreement with data[65];

$$M_{D^*}^2 - M_D^2 \approx 0.55\text{GeV}^2, \qquad M_{B^*}^2 - M_B^2 \approx 0.48\text{GeV}^2. \tag{3.1.44}$$

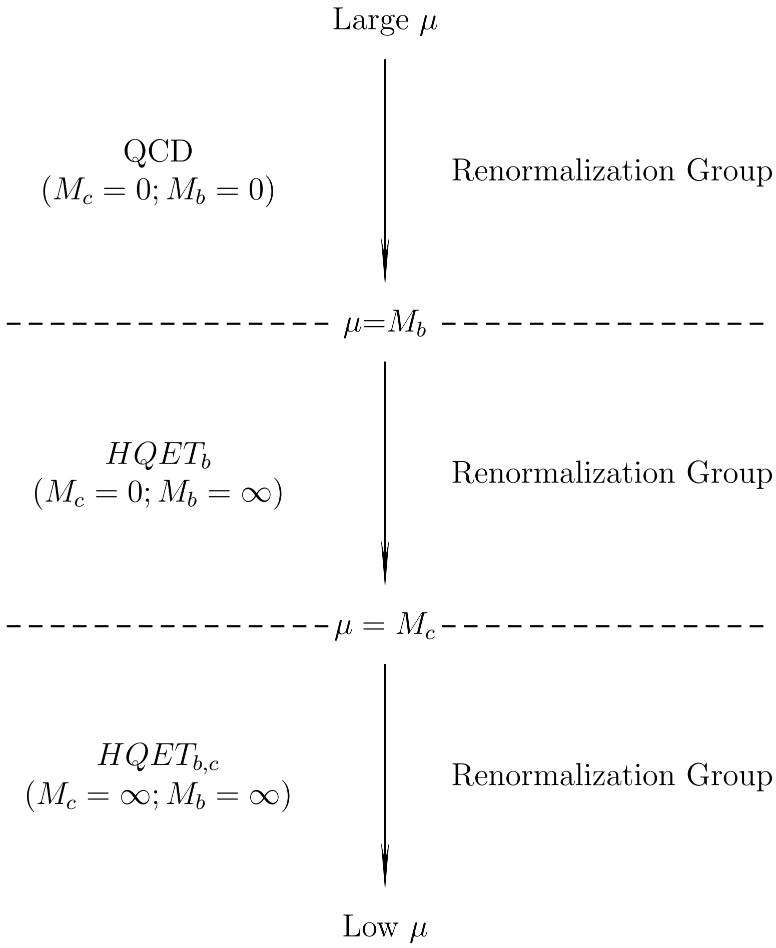


Figure 3.3: Evolution from high to low scales in heavy-quark physics.

3.1.4 Renormalization and Matching

The general procedure to evolve down in energy is shown in Fig(3.3). One starts with the full QCD theory at high scale, where the b quark can be considered light (rather mass less in first approximation). Using the renormalization group, one goes down to $\mu = m_b$, where the small component of b-quark field is integrated out, and matching between QCD and HQET takes place. Below m_b, one makes use of HQET for the b- quark until the scale $m_c$is reached. One then performs the further integration of small components also for charm quark, and changes to a different HQET where both b and c quarks are considered as heavy.
The numerical accuracy of the HQET predictions will be different in two HQETs, owing to the different masses of the bottom and charm quarks. While the $1/m_b$ expansion is expected to work well, corrections of $\mathcal{O}(1/m_c)$ could be large in many cases.

3.2 Soft-Collinear Effective Field Theory

Soft-Collinear Effective Theory (SCET) is another form of the effective theory derived from QCD. SCET describes the dynamics of highly energetic particles moving close to the light-cone and interacting with the background field of soft quanta. In HQET all components of residual momentum k^μ given in Eq.(3.1.1)are assumed to be of order Λ_{QCD}. In decay of heavy quark one may have a kinematical situation in which the light degree of freedom carry a large amount of energy in the rest frame of heavy quark resulting on three different energy scale for light quark with large energy.

SCET Lagrangian can be formulated by considering light particles moving as a jet close to the light-cone direction n^μ. Dynamics of these particles is best described by light-cone coordinates

$$k^\mu = \frac{k_+}{2} n_+^\mu + k_\perp^\mu + \frac{k_-}{2} n_-^\mu \tag{3.2.1}$$

with $k_+ = n_+ \cdot \vec{k}$,$k_- = n_- \cdot \vec{k}$. With velocity of heavy quark in term of light cone vectors n_+ and n_- is given as

$$v = \frac{1}{2}\left(n_+ + n_-\right), \qquad n_\pm^2 = 0, \qquad (n_- \cdot n_+ = 2) \tag{3.2.2}$$

which satisfies the relations $n_\pm^2 = 0, (n_- \cdot n_+ = 2)$,and the metric tensor can be decomposed as

$$g^{\mu\nu} = \frac{1}{2}\left(n_+^\mu n_-^\nu + n_-^\mu n_+^\nu\right) + g_\perp^{\mu\nu}. \tag{3.2.3}$$

For end point regions where the energies of the particle are close to maximal values different light cone components are widely separated, one may have $k_- \sim m_b$ very large while $k_\perp$ and k_+ is small. Taking a small dimensionless parameter λ in order to define a consistent power counting final-state invariant mass can be written as

$$k^2 = k_- k_+ + k_\perp^2 \sim \mathcal{O}(\lambda^2 m_b^2). \tag{3.2.4}$$

Thus the light-cone momentum components of collinear particle scale like $k_c = m_b(1,\lambda,\lambda^2)$. This collinear quark can emit either a gluon collinear to large momentum direction or gluon with momentum scaling $k_{us} = m_b(\lambda^2,\lambda^2),\lambda^2$. For scales above the typical off-shellness of the collinear degrees of freedom, $k_c^2 \sim (m_b\lambda^2)$, both these gluon modes are required to correctly reproduce all infrared physics of QCD.

SCET Lagrangian can be constructed in a similar way as for HQET. One may start from the Lagrangian of a mass less quark q,

$$\mathcal{L} = \bar{q} i \not{D} q, \tag{3.2.5}$$

where D denotes the covariant derivative of QCD. With the above kinematical assumptions, one may use the two light-cone vectors to define projectors

$$\mathcal{P} = \frac{1}{4}\not{n}_-\not{n}_+, \qquad \mathcal{Q} = \frac{1}{4}\not{n}_+\not{n}_-, \text{where} \qquad \mathcal{P} + \mathcal{Q} = 1. \tag{3.2.6}$$

In a similar way to that used in HQET, one can split the quark field q into two components

$$\xi = \mathcal{P}q, \qquad \eta = \mathcal{Q}q. \tag{3.2.7}$$

Likewise

$$\not{D} = \frac{1}{2}\not{n}_+(in_-D) + \frac{1}{2}\not{n}_-(in_+D) + \not{D}_\perp. \tag{3.2.8}$$

Inserting this back and using $\not{n}_-\xi = 0 = \not{n}_+\eta$, one obtains

$$\mathcal{L} = \frac{1}{2}\bar{\xi}\not{n}_+(in_-D)\xi + \frac{1}{2}\bar{\eta}\not{n}_-(in_+D)\eta + \bar{\eta}i\not{D}_\perp\eta + \bar{\eta}i\not{D}_\perp\xi \tag{3.2.9}$$

Now since $(in_+D) \sim m_b$ and $(in_+D) \sim \lambda^2 m_b$, on the basis of the power counting η field should be integrated out. This can be done by integrating over the Green's functions, written as a functional integral over the quark field. Performing this integration corresponds to using the equation of motion

$$\eta = -\frac{1}{in_+.D + i\epsilon}\frac{\not{n}_+}{2}iD_\perp\xi \tag{3.2.10}$$

and inserting it back into Lagrangian

$$\mathcal{L} = \frac{1}{2}\bar{\xi}\not{n}_+(in_-.D) - \bar{\xi}i\not{D}_\perp\frac{1}{in_+.D + i\epsilon}\frac{\not{n}_+}{2}iD_\perp\xi \tag{3.2.11}$$

This resulting Lagrangian is still completely equivalent to that of full QCD, but it is now expressed in terms of the collinear quark field. However it is still non-local and will become local only after expansion. To perform this expansion, one need to identify the large contribution in the quantity $in_+ \cdot D$. In order to do so, the gluon field A splits into collinear contribution A_c and ultrasoft contribution A_{us}

$$in_+D = in_+\partial + gn_+A_c + gn_+A_{us} = in_+D_c + gn_+A_{us} \tag{3.2.12}$$

where collinear covariant derivative $iD_c = i\partial + gA_c$ containing the collinear gluon field. In order to have complete Lagrangian, one need to do the decomposition of the gluonic part of the QCD Lagrangian. Also one need to include the ultrasoft quarks scales like $m_b\lambda^2$, which appear as spectator quark in heavy hadron. It is to be noted that in the leading order Lagrangian, the only coupling to ultrasoft degrees of freedom is the coupling from (in_-D) [72] to the collinear quarks. A similar coupling appears in the gluonic sector, where one has an n_-A_{us} coupling of ultrasoft gluons to collinear gluons. This observation is the basis of the factorization theorems being investigated intensively.

SCET Lagrangian presented above, not only has a global helicity spin symmetry but also has a powerful set of gauge symmetries [75]. Specifically the collinear and ultrasoft fields each have their own gauge transformation that leave the Lagrangian invariant. Collinear gauge transformations are the subset of QCD gauge transformations where $\partial^\mu U(x) \sim m_b\,(\lambda^2, \lambda, 1)$, and the ultrasoft gauge transformations are those where $\partial^\mu V(x) \sim m_b\lambda^2$. The invariance under each these transformations is a manifestation of scales of order m_b or greater having been removed from the theory, since any gauge transformation that would change a ultrasoft gluon into a collinear

Momentum Scaling	Terminology I	Terminology II
(1,1,1)	hard	hard
(λ,λ,λ)	soft	semi-hard
(1,λ, λ^2)	collinear	hard-collinear
(λ^2,λ^2,λ^2)	ultrasoft	soft
(1,λ^2,λ^4)	ultracollinear	collinear

Table 3.1: Terminology for the various momentum modes relevant to exclusive B decays. The momentum components are given as (n_+k,$k_\perp$,n_-k), but mass dimension has to be restored by multiplying appropriate factors of m_b. Two different terminologies for the same momentum modes are used. In physical units λ is of order $(\Lambda/m_b)^{1/2}$, where Λ is the strong-interaction scale.

gluon would imply a boost of order m_b.
The scaling chosen to derive SCET Lagrangian is referred as SCET$_I$ and is more commonly used to discuss the inclusive decays. For the study of exclusive channels, SCET$_{II}$ is used which has different power counting and is shown in column II in the Table (3.1). As exclusive decay of $B \to D(D^*)l\nu$ is under consideration so from now on SCET is referred as SCET$_{II}$.

3.3 Factorization and SCET

SCET has become a standard tool to study the factorization [66][75] of short- and long- distance effects in processes involving low-energetic particles and high-energetic/low-virtuality modes. The ability to provide precise theoretical predictions for high-energy processes are based on the factorization. For strong interactions especially if the factorization holds, the effects of heavy particles and /or highly virtual radiative corrections can be calculated in perturbative QCD, while the long-distance physics of light quarks and gluons can be encoded in hadronic matrix element of composite operators, which then can studied using non-perturbative methods. One of the important and general feature of factorization is the appearance of a factorization scale μ that relates the infrared (IR) divergences, appearing in loop corrections to short-distance amplitudes/cross sections, ad the ultraviolet (UV) divergences of composite operators defining the long-distance matrix elements, such that the scale dependence cancels to any given order in perturbation theory.

SCET applications in exclusive B decays reveals some new aspects. It is realized that the decay into few light energetic hadrons (with mass $m^2 \sim \mathcal{O}(\Lambda^2_{QCD})$) is power-suppressed compared to the production of a generic jet (with mass $m_X^2 \sim \mathcal{O}(\Lambda_{QCD} m_b)$), since it requires a particular fine-tuning in the phase space of B-meson spectator system. A related subtlety arise from endpoint divergences which prevent the complete (perturbative) factorization of soft and collinear modes (with small invariant mass$\sim m^2$). Factorization theorems for exclusive heavy to heavy amplitudes thus takes the generic form [73][74]

$$\mathcal{A}_i\left(B \to MM'\right) = \xi_M \cdot T_i^I \otimes \phi_{M'} + T_i^{II} \otimes \phi_B \otimes \phi_M \otimes \phi'_M + \cdots , \qquad (3.3.1)$$

where M, M' denoted mesons in the final state. $T_i^{I,II}$, are short-distance function, where the T_i^{II} can be further factorize into the hard and an exclusive jet function (including spectator scattering), but the T_i^I do not. Further more ξ_M denotes a universal form factor for $B \to M$ transitions, and $\phi_{B,M}$ are light-cone distribution amplitudes for initial and final hadrons. $\otimes$ denotes convolution of kernel T_i^I,T_i^{II} with the light-cone distribution amplitudes $\phi_{B,M}$. For exclusive $B \to D(D^*)l\nu$ the above factorization formula take the form

$$f_i(q^2) = C_i \xi_R(E) + \Phi_B \otimes T_i \otimes \Phi_f \tag{3.3.2}$$

where $\xi_R(E)$ is the soft part of the form factor which obey the symmetry relations, T_i is the hard-scattering kernel and Φ_B and Φ_f are the light-cone distribution amplitudes for the B and the final state meson f. It is also to be noted that the short-distance functions are soft form factors, which obeys the large recoil symmetries. For the heavy-heavy case the collinear effects are irrelevant and remaining hard and soft interactions factor into short-distance coefficient and the Isgur-Wise form factor similar to the first term on the right-hand side of Eq.(3.3.1). In order to do the factorization of form factors relevant momentum modes should be studied. The detail study of different momentum modes is done in next section.

3.4 The scalar "photon" vertex

The existence of various modes follows from the assumption that the external momenta of scattering amplitudes for exclusive B decays at large momentum transfer are soft and collinear. One find the three characteristic virtualities m_b^2,$m_b\Lambda$ and Λ^2 by combining external momenta. For instance, m_b^2 is obtained by adding and squaring a heavy quark and collinear momenta, or by squaring the heavy quark momentum. The intermediate virtuality is typical for interactions of collinear gluons or light quarks with the soft gluons or light quarks, while Λ^2 arises in the self-interactions of collinear or soft modes.

Although SCET is obtained after integrating out hard modes of virtuality m_b^2 but it still contains two types of soft modes, called "semi-hard"(virtuality of order $m_b\Lambda$) and "soft". The semi-hard modes can be integrated out perturbatively, but it appears that semi-hard loop integrals always vanishes in dimensional regularization [77], so they can be ignored. The theory also contains two types of collinear modes, called "hard-collinear" and "collinear" according to there virtualities.

In order to understand hard-collinear and collinear modes in detail and to see how the factorization of collinear and soft modes introduces endpoint singularities which then can be canceled by taking the sum of all terms, one can start with a simple scalar integral [78]

$$I = \int [dk] \frac{1}{(k-l)^2[k^2-m^2][(p-k)^2-m^2]} \tag{3.4.1}$$

which occurs as a one-particle-irreducible subgraph in the correction to the radiative decay $B \to \gamma l\nu$ as shown in Fig(3.4). With light quark having mass m ans

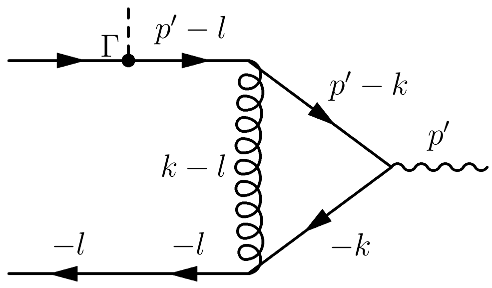


Figure 3.4: Photon vertex correction to $\bar{B} \to \gamma l \nu$. Γ denotes the weak $b \to c$ decay vertex. The vertex integral is considered with all lines simplified to scalar propagators and all vertex factors set to 1.

integration measures to be defined as

$$[dk] = \frac{(4\pi^2)}{i}\left(\frac{\mu^2 e^{\gamma E}}{4\pi}\right)^{\epsilon}\frac{\mathrm{d}^d k}{(2\pi)^d}$$
$$= \mu^{2\epsilon} e^{\epsilon\gamma E}\frac{\mathrm{d}^d k}{i\pi^{d/2}} \qquad (d = 4 - 2\epsilon)\,. \tag{3.4.2}$$

with

$$\mathrm{d}^d k = \mathrm{d}n_- k \mathrm{d}n_+ k \mathrm{d}^{d-2} k_\perp \tag{3.4.3}$$

The integral I is ultraviolet and infrared finite, but dimensional regularization will be needed to construct the expansion. The external momentum of the vertex subgraph are a collinear photon momentum $p' = (n_+p', p'_\perp, n_-p') \sim (1,0,0)$ with $p'^2 = 0$, a soft light quark momentum $l \sim (\lambda^2,\lambda^2,\lambda^2)$ with $l^2 = m^2$ and a hard-collinear light quark momentum $p' - l \sim (1,\lambda^2,\lambda^2)$ with virtuality λ^2. The two invariants are $2p' \cdot l \sim \lambda^2$ and $m^2 \sim \lambda^4$, so I must be a function of the small dimensionless ratio $m^2/\left(2p' \cdot l\right)$. After a straightforward calculation one gets

$$I = \frac{1}{2p' \cdot l}\left(\mathrm{Li}_2\left(-\frac{2p'.l}{m^2}\right) - \frac{\pi^2}{6}\right) = \frac{1}{2p' \cdot l}\left\{\frac{1}{2}\ln^2\frac{m^2}{2p' \cdot l} + \frac{\pi^2}{3} + \cdots\right\}, \tag{3.4.4}$$

where after second equality higher order terms in $m^2/\left(2p' \cdot l\right)$ are neglected.

3.4.1 Expansion by momentum regions

The expansion of I is done by identifying the momentum configurations that give non-vanishing contributions to the integral in dimensional regularization. The integrand are then expanded in each region. To find the relevant momentum regions it is first assumed that loop momentum scales as $k \sim (\lambda^n,\lambda^n,\lambda^n)$ for some n and expand the integrand accordingly. For instance, if k is hard, $n = 0$, one finds integrals of type

$$I = \int [dk] \frac{1}{[k^2]^a[-2p' \cdot k]^b} \times \text{polynomial}, \tag{3.4.5}$$

which vanishes in dimensional regularization, since only possible invariant $p' = 0$ and there is no external invariant of order 1. Proceeding for different n one founds that

only $n = 2$, contributes which is called as soft momentum region. As there are external lines with large momentum and small virtuality, one should also consider loop momentum configurations, where n_+k has largest component. It means that one can take $n_+k \sim \lambda^n$ and $k^2 \sim \lambda^{2m}$ with $m < n$, expand the integrand and determine the integrals that do not vanish. It is also to be noted that some integrals vanishes independent of any regularization as all poles lie in one of the complex half-planes. The process involves taking in all propagators in integral, then using the different power counting depicted in Table.(3.1). Choosing the leading order contributions in different components of the propagators, performing the n_-k integration by contour methods. After that one needs to see the limits for which integral diverges and has to do the integration using appropriate methods.
Before going into the detailed calculations let us first consider the momentum modes which appears in the hadronic wave functions of B- and D- meson states. In the B-meson rest frame, the heavy quark is on-shell up to a residual momentum of order Λ_{QCD},

$$b - \text{quark}: \qquad p_b^\mu = m_b v^\mu - l^\mu, \tag{3.4.6}$$

Consequently, the spectator quarks and gluons in the B-meson are characterized by energies and momenta of order $\Lambda_{QCD} = \lambda^2 m_b$ will refer to as "soft".
The same situation applies for the D-meson in its rest frame. However, from the perspective of the decaying B-meson, the constituents of the D-meson should be written as

$$c - \text{quark}: \qquad p_c'^\mu = m_c v'^\mu - k'^\mu \tag{3.4.7}$$

However, from the perspective of the decaying B-meson, the constituents of the D-meson should be written as

$$n_+k' \frac{n_-^\mu}{2} - k_\perp'^\mu - n_-k' \frac{n_+^\mu}{2} \tag{3.4.8}$$

and thus have the momentum scaling

$$(n_+k', k'_\perp, n_-k') \sim m_b \left(\lambda, \lambda^2, \lambda^3\right) \tag{3.4.9}$$

this configuration can be called as "soft". For the case under consideration of $\bar{B} \to D(D*)l\bar{\nu}$ the integral given in Eq.(3.4.1) takes the form

$$I = \int [dk'] \frac{1}{(k'-l)^2 \left[k'^2 - m^2\right] \left[(p'-k')^2 - m_c^2\right]} \tag{3.4.10}$$

with mass of charm-quark scale likes $m_c \sim \lambda m_b$.

The hard-collinear region

For the case of hard-collinear region one takes $k' \sim (1,\lambda,\lambda^2)$, $l \sim (\lambda^2,\lambda^2,\lambda^2)$ and $p' \sim (1,0,\lambda^2)$ with $p'^2 - m_c^2 \sim \lambda^3$. Expansion of the propagators given in Eq.(3.4.10) gives the following contributions for the leading hard-collinear integral

$$I_{hc} = \int [dk'] \frac{1}{\left[k'^2 - (n_+k')(n_-l)\right] \left[k'^2\right] \left[k'^2 - (n_-k')(n_+p') - (n_+k')(n_-p')\right]} \tag{3.4.11}$$

where $k'^2 = n_-k'n_+k' + k'^2_\perp$. For contour integration one needs to identify the poles and the position of these poles. There are three different regions namely $n_+k' < 0$,$0 < n_+k' < n_+p'$ and $n_+k' > n_+p'$. Different values of the n_-k' obtained are

$$\begin{aligned} n_-k' &= \frac{-k'^2_\perp + n_+k' \cdot n_-l + i\epsilon}{n_+k'} \\ &= \frac{-k'^2_\perp + i\epsilon}{n_+k'} \\ &= \frac{-k'^2_\perp + n_+k' \cdot n_-p' + i\epsilon}{n_-k' - n_+p'} \end{aligned} \tag{3.4.12}$$

Given with the different regions it is easy to locate the position of the poles. With n_+p' is related to the energy of outgoing particle so it has to be greater then 0 and $n_-p' \sim m_c^2/n_+p'$. The contour can be closed in the upper half plane and residual $\frac{-k'^2_\perp + n_+k' \cdot n_-p' + i\epsilon}{n_-k' - n_+p'}$ could be taken. Hence one gets the integral after contour integration

$$I_{hc} = \frac{(\mu^2 e^{\gamma E})^\epsilon}{i\pi^{d/2}} \int_0^\infty \mathrm{d}n_+k' \int \mathrm{d}^{d-2}k'_\perp \left[k'^2_\perp + n_+k'(\frac{-k'^2_\perp + n_+k' \cdot n_-p'}{n_+k' - n_+p'}) - n_+k' \cdot n_-l\right]^{-1} \left[k'^2_\perp + n_+k'(\frac{-k'^2_\perp + n_+k' \cdot n_-p'}{n_+k' - n_+p'})\right]^{-1} \left[(\frac{1}{n_+k' - n_+p'})\right]^{-1} \tag{3.4.13}$$

After substituting $n_+k' \to un_+p'$ and $n_-p' \sim m_c^2/n_+p'$ and doing the appropriate limit change one lefts with a simplified form of the above integral as

$$I_{hc} = \frac{(\mu^2 e^{\gamma E})^\epsilon}{i\pi^{d/2}} \int_0^1 \mathrm{d}u \int \mathrm{d}^{d-2}k'_\perp \frac{(u-1)}{(-k'^2_\perp + m_c^2u^2)\,(-k'^2_\perp + m_c^2u^2) - (n_-l)(n_+p')u\,(u-1))} \tag{3.4.14}$$

By using the standard techniques for the n_+k' and $k'_\perp$ integration, one arrives at the result as

$$I_{hc} = -\frac{1}{(n_-l)(n_+p')}\left\{\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\left(\log\frac{\mu^2}{m_c^2} + 2\log\frac{(n_-l)(n_+p')}{m_c^2}\right) + \frac{1}{4}\log^2\frac{\mu^2}{m_c^2} - \log\frac{\mu^2}{m_c^2}\log\frac{(n_-l)(n_+p')}{m_c^2} + \frac{1}{2}\log^2\frac{(n_-l)(n_+p')}{m_c^2} - \mathrm{Li}_2\left(1 - \frac{m_c^2}{(n_-l)(n_+p')}\right) + \frac{\pi^2}{24}\right\} \tag{3.4.15}$$

The soft(HQET) region

As discussed earlier that the collinear effects are irrelevant but here because of the boost discussed in Eq.(3.4.9),there is non-vanishing momentum region which is called as soft(HQET). The momentum configuration used for $k' \sim (\lambda,\lambda^2,\lambda^3)$, $l \sim (\lambda^2,\lambda^2,\lambda^2)$ and $p' \sim (1,0,\lambda^2)$. Soft(HQET) and soft integrals are not well-defined separately in dimensional regularization. It is because of the fact that dimensional regulator is attached to the transverse momentum components. The additional divergences arising from n_+k' or n_-k' integrations may not regularized. This occurred in the method of expansion by regions to collinear integrals discussed in [79]. As in [79] [80] an additional "analytical" regularization is done by substituting

$$\frac{1}{[(k'-l)^2]} \to \frac{[-\nu^2]^\delta}{[(k'-l)^2]^{1+\delta}}, \tag{3.4.16}$$

where ν is a parameter with mass dimension one. The leading soft(HQET) integral then can be given as

$$I_{sh} = \int [dk] \frac{[-\nu^2]^\delta}{[-(n_+k)(n_-l)]^{1+\delta}\,[k^2-m^2]\,[-(n_-k)(n_+p)-(n_+k)(n_-p)]} \tag{3.4.17}$$

The contour then can be closed in the lower half plane and one can pick up the pole at $(-k_\perp'^2 + m^2 - i\epsilon/n_+k')$ for $0 < n_+k' < n_+p'$. After the contour integration and making substitution $n_+k' \to un_+p'$ a simplified form is obtained

$$I_{sh} = -\frac{(\mu^2 e^{\gamma E})^\epsilon}{i\pi^{d/2}} \int_0^1 \mathrm{d}u \int \mathrm{d}^{d-2}k'_\perp \frac{[-\nu^2]^\delta}{(-(n_+p)(n_-l)u)^{1+\delta}\,(-k_\perp'^2 + m^2 + m_c^2u^2)} \tag{3.4.18}$$

The final result reads as

$$\begin{aligned} I_{sh} = -\frac{1}{(n_+p')(n_-l)} \Bigg\{ & \frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\log\frac{\mu^2}{m_c^2} + \frac{1}{2}\log\frac{\mu^2}{m_c^2}\log\frac{\mu^2}{m^2} - \frac{\pi^2}{24} \\ & -\frac{1}{4}\log^2\frac{\mu^2}{m^2} + \left(\frac{1}{\epsilon} + \log\frac{\mu^2}{m^2}\right)\left(-\frac{1}{\delta} - \log\frac{\nu^2}{(n_+p')(n_-l)}\right)\Bigg\} \end{aligned} \tag{3.4.19}$$

the pole at $\epsilon = 0$ comes from the $k'_\perp \to \infty$ (for any n_+k') in the transverse momentum integral. The additional singularity at $\delta = 0$ is an "endpoint divergence", which arise from the singularity at $n_+k' \to 0$ for a transverse momentum. This does not correspond to any singularity of the hard-collinear integral. Since n_-k' become large compared to λ^4, when n_+k' becomes small, the end point singularity is related to a momentum configuration, where quark with momentum k' becomes soft. In particular, as the end point divergence occurs for any $k_\perp \sim \lambda^2$ it must be canceled by a momentum region with $k'_\perp \sim \lambda^2$. Also to be noted that the integral depends non-analytically on the soft external momentum component n_-l. This is surprising, since one would have expected factorization of the soft and soft (HQET) modes, so that the soft (HQET) integrals could depend only analytically on n_-l, and soft integrals could depend only analytically on n_+p'. Indeed, this could be the case in dimensional regularization, where factor $1/n_-l$ in Eq.(3.4.17) could be pulled out of the integral. However, the integral is not well-defined in dimensional regularization. The breakdown of the naive soft (HQET)-soft factorization is hence a consequence of the need to introduce a different regularization, here which are chosen as analytic.

The soft region

In soft region the gluon propagator and the light quark propagator with momentum $-k'$ are soft with virtuality of order λ^4. Th quark with momentum $p'-k'$ is hard-collinear with virtuality λ^2. The leading soft integral is

$$I_s = \int [dk] \frac{1}{[(k'-l)^2]^{1+\delta}\,[k'^2-m^2]\,[-(n_-k')(n_+p')]} \tag{3.4.20}$$

Here the n_+k' integration is performed first. Assuming $n_-l > 0$, the contour is closed in the lower half plane and the pole at $(-k'^2_\perp + m^2 - i\epsilon)/n_-k'$ is picked up for the region $0 < n_-k' < n_-l$. After contour integration and using substitution $n_-k' \to un_-l$ one gets

$$I_s = \frac{(\mu^2 e^{\gamma E})^\epsilon}{i\pi^{d/2}} \int_0^1 \frac{du \int d^{d-2}k'_\perp [-\nu^2]^\delta}{(k'^2_\perp + m^2(1-u)^2)^{1+\delta}\, u^{1-\delta}} \tag{3.4.21}$$

There is a singularity for $k'_\perp \to \infty$ for any n_-k'. The pole at $\delta = 0$ is an endpoint divergence from $n_-k' \to 0$ for any $k'_\perp$. This implies that n_+k' becomes large for fixed $k_\perp \sim \lambda^2$, and hence the quark with momentum k' becomes collinear. In soft region the transverse momentum and longitudinal momentum integrals do not factorize, and there is also a divergence when $k'_\perp \to \infty$ and $n_-k' \to 0$ simultaneously, which corresponds to a double pole in the hard-collinear integral. Solving the above integral one gets

$$I_s = -\frac{1}{(n_+p')(n_-l)} \left(\frac{\mu^2 e^{\gamma E}}{m^2}\right)^\epsilon \Gamma(\epsilon) \left(\frac{m^2}{\nu^2}\right)^{-\delta} \frac{1}{\delta} \frac{\Gamma(\delta+\epsilon)\Gamma(1-2\delta-2\epsilon)}{\Gamma(\epsilon)\Gamma(1-2\delta-2\epsilon)}. \tag{3.4.22}$$

As the hard-collinear contribution is not regularized analytically, the correct procedure is to expand first in δ and then in ϵ. the pole in δ cancels with the soft (HQET) contribution before expansion in ϵ. However after performing both expansions result obtained is

$$\begin{aligned} I_s = -\frac{1}{(n_+p')(n_-l)} \Bigg\{ & \left(\frac{1}{\epsilon} - \ln\frac{m^2}{\mu^2}\right)\left(\frac{1}{\delta} - \ln\frac{m^2}{\nu^2}\right) \\ & -\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{m^2}{\mu^2} - \frac{1}{2}\ln^2\frac{m^2}{\mu^2} + \frac{5\pi^2}{12} \Bigg\} \end{aligned} \tag{3.4.23}$$

The expansion in δ has generated a double pole in ϵ.
There are few other regions as well like semi-hard region $k \sim (\lambda,\lambda,\lambda)$ and the semi-collinear region $k \sim (\lambda,\lambda^{3/2},\lambda^2)$ but they vanish in dimensional regularization.

Adding up

The singularity in δ cancels in the sum of the soft(HQET) and soft integral, after adding into hard-collinear integral and substituting $n_-p' \sim m_c^2/n_+p'$ the singularity in ϵ also cancels and one obtain a very simple relation given as

$$\begin{aligned} I_{hc} + I_{hs} + I_s = \; & \frac{1}{(n_+p')(n_-l)} \Bigg\{ \frac{1}{4}\ln^2\frac{m_c^2}{m^2} - \frac{1}{2}\ln^2\frac{m^2}{(n_+p')(n_-l)} \\ & + \mathrm{Li}_2\left(1 - \frac{m_c^2}{(n_+p')(n_-l)}\right) - \frac{5\pi^2}{12} \Bigg\} \end{aligned} \tag{3.4.24}$$

and it is in total agreement with the expansion in Eq.(3.4.4). In general one must consider the different momentum region. In scalar integral given in Eq.(3.4.1) all three regions con tribute already to the leading term in the expansion. In QCD the photon-vertex integral contains a numerator proportional to n_-k' which suppresses the collinear region by a factor of λ^2 relative to the hard-collinear and soft region. For this reason it is sufficient to consider only hard-collinear and soft configurations in the factorization theorem foe $B \to \gamma l\nu$ at leading power in $1/m_b$, as has been done in [81][85][82]. Hard-collinear modes are perturbative and can be integrated out, resulting in hard-scattering kernels. Soft and collinear modes have virtuality $\lambda^4 \sim \Lambda^2$, and cannot be treated in perturbation theory. The $1/m_b$ suppression of the collinear contribution in QCD implies that the hadronic structure of the photon in a sub-leading effect in $B \to \gamma l\nu$ decays.

3.5 Hadronic matrix element

In order to compute the physical quantities one needs to evaluate the hadronic matrix elements of HQET operators. The matrix elements of flavor-changing currents $\bar{q}\Gamma b$ are important strong interaction parameters in low-energy weak-interactions processes. The strong interaction dynamics of semi leptonic B decays is encoded in these form factors. A better understanding of such quantities improves the accuracy of the extraction of CKM matrix parameters from experimental data, and of the searches for new phenomena in the flavor-changing processes.

3.5.1 Matrix elements in HQET

The relations among different matrix elements in HQET can be derived by using the fallout and spin symmetries. The standard relativistic normalization for hadronic states is [76]

$$\langle H(p')|H(p)\rangle = 2E_p \left(2\pi^3\right) \delta^3 \left(p - p'\right), \tag{3.5.1}$$

With $E_p = \sqrt{|p|^2 + m_H^2}$. States having the above normalization have mass dimension -1. In HQET, hadron states are labeled by a four-velocity v^μ and a residual momentum k^μ satisfying $v.k = 0$. These states are defined by using the HQET Lagrangian in $m_Q \to \infty$. They differ from full QCD states by $1/m_Q$ corrections and the normalization factor. The normalization convention in HQET is

$$\langle H(v',k')|H(v,k)\rangle = 2v^0\delta_{vv'} \left(2\pi^3\right) \delta^3 \left(k - k'\right). \tag{3.5.2}$$

Possible spin labels are suppressed in Eq.(3.5.1) and Eq.(3.5.2). The split between the four-velocity v^μ and the residual momentum k^μ is somewhat arbitrary, and hence is a freedom to redefine v^μ by an amount of order Λ_{QCD}/m_Q while changing k^μ by a corresponding amount of order Λ_{QCD}. This is called re-parametrization invariance. The hadronic states are redefined as mass-independent meson states as

$$|H(v)\rangle \equiv \frac{1}{m_H}|H(p)\rangle. \tag{3.5.3}$$

The implications of the HQET symmetries can be derived in a simple way by using a covariant tensor representation of the states with definite transformation properties under the Lorentz group and the heavy-quark spin-fallout symmetry [83][68][67]. For better understanding one can consider the lowest $Q\bar{q}$ multiplet ($s_l = 1/2$), which contains a doublet of degenerate spin-zero and spin-one mesons $H \equiv [P(0^-),V(1^-)]$.

Knowing their transformation symmetry properties, one can built appropriate wave functions to represent the states:

$$\begin{aligned} P(v) &\propto \quad \langle 0|h_v^{(Q)}\bar{q}|P(v)\rangle \sim -P_+\gamma_5, && (3.5.4)\\ V(v,\epsilon) &\propto \quad \langle 0|h_v^{(Q)}\bar{q}|V(v,\epsilon)\rangle \sim P_+\not{\epsilon}, && (3.5.5)\end{aligned}$$

where ϵ is a polarization of the vector meson ($\epsilon^* \cdot \epsilon = -1, v \cdot \epsilon = 0$). Since the two states are related by the symmetry transformations, one can introduce a combined wave function $\mathcal{M}(v)$ that represents both $P(v)$ and $V(v,\epsilon)$:

$$\begin{aligned} \mathcal{M}(v) &\equiv \qquad P_+\left[-a\gamma_5 + \textstyle\sum_\epsilon b_\epsilon \not{\epsilon}\right], \\ \bar{\mathcal{M}}(v) &\equiv \quad \gamma^0 H^\dagger(v)\gamma^0 = \left[a^*\gamma_5 + \textstyle\sum_\epsilon b_\epsilon^* \not{\epsilon}^*\right] P_+. && (3.5.6)\end{aligned}$$

Because of the positive-energy projector these states satisfy $\not{v}\mathcal{M}(v) = \mathcal{M}(v)$, $\bar{\mathcal{M}}(v)\not{v} = \bar{\mathcal{M}}(v)$,$\mathcal{M}(v) = P_+\mathcal{M}(v)P_-$ and $\bar{\mathcal{M}}(v) = P_-\bar{\mathcal{M}}(v)P_+$. The coefficient a and b_ϵ are labels which indicate a particular meson state ($a = 1$,$b_\epsilon = 0$ for the pseudoscalar state $a = 0$,$b_\epsilon = \delta_{\epsilon\epsilon_0}$ for vector state with polarization ϵ_0).

In order to compute the hadronic matrix element of a given operator $\mathcal{O}$, one replaces the hadronic states by appropriate wave functions and builds the most general object with the same symmetry structure as $\mathcal{O}$. For instance, the norm of the meson states can be evaluated through

$$\begin{aligned} \langle M(v)|M(v)\rangle \quad &= \mathrm{tr}\left[\bar{\mathcal{M}}(v)\mathcal{M}(v)\left(A + B\not{v} + \cdots\right)\right] \\ &= N\mathrm{tr}\left[\bar{\mathcal{M}}(v)\mathcal{M}(v)\right] = -2N\left(|a|^2 + \sum_\epsilon |b_\epsilon|^2\right). && (3.5.7)\end{aligned}$$

All possible invariant combinations $(1,\not{v},\not{v}\not{v},\cdots)$ should be included. Since $\mathcal{M}(v)\not{v} = -\mathcal{M}(v)$ and $\not{v}\not{v} = 1$, in this case all structure reduce to identity operator. Thus, there is only an arbitrary factor N which fixes the global normalization. The results shows that the relative normalization of the pseudoscalar and vector states in Eq.(3.5.6) is correct.

Considering the matrix element of a quark current $\bar{h}_{v'}^{(Q')}\Gamma h_v^{(Q)}$, which changes a heavy quark Q into another heavy quark Q'. Lorentz covariance forces the amplitude to be proportional to $\bar{\mathcal{M}}'(v')\Gamma\mathcal{M}(v)$. This structure should be multiplied by an arbitrary function of all Lorentz invariants

$$X(v,v') = X_0(v,v') + X_1(v,v')\not{v} + X_2(v,v')\not{v}' + X_3(v,v')\not{v}\not{v}' \qquad (3.5.8)$$

Within the trace tr $\left[\bar{\mathcal{M}}'(v')\Gamma\mathcal{M}(v)X(v,v')\right]$, the $\not{v}$ operators can be eliminated, using the projection properties of the meson wave functions

$$X(v,v') \to X_1 - X_2 - X_3 + X_4 \equiv -\xi(v \cdot v'). \qquad (3.5.9)$$

Therefore

$$\begin{aligned}\langle \bar{\mathcal{M}}'(v')\bar{h}_{v'}^{(Q')}\Gamma h_v^{(Q)}|\mathcal{M}(v)\rangle \quad &= -\xi(v\cdot v')\mathrm{tr}\left[\bar{\mathcal{M}}'(v')\Gamma\mathcal{M}(v)\right] \\ &= -\xi(v\cdot v')\mathrm{tr}\left[\frac{1+\not{v}'}{2}\Gamma\frac{1+\not{v}}{2}\right. \\ &\left(-aa'^{*}+\sum_{\epsilon\epsilon'}b_\epsilon b'^{*}_{\epsilon'^{*}}\not{\epsilon}\not{\epsilon}'^{*}\right. \\ &\left.\left.-a\sum_{\epsilon'}b'^{*}_{\epsilon'^{*}}\gamma_5\not{\epsilon}'^{*}+a'^{*}\sum_{\epsilon}b_\epsilon\not{\epsilon}\gamma_5\right)\right]. \quad (3.5.10)\end{aligned}$$

This equation summarizes in a compact form the consequences of the HQET symmetries. All current matrix elements are given in terms of same function $\xi(v\cdot v')$, which is called as Isgur-Wise function. Taking the appropriate a and b_ϵ labels, one can easily derives the explicit expressions for the matrix elements which are relevant in semileptonic decays.

3.6 Exclusive semi-leptonic decays

Semi leptonic decays of B mesons have received a lot of attention in recent years. The decay channel $\bar{B}\to D^{*}l\bar{\nu}$ has the largest branching fractions of all B-meson decay modes. Schematically, a semi-leptonic decay process is shown in Fig(3.5). The strength of the $b\to c$ transition vertex is governed by the element V_{cb} of the CKM matrix. The form factors for $\bar{B}$ decays are given by the Lorentz decompositions of bi linear quark current matrix elements:

$$\begin{aligned}\langle P(p')|\bar{q}\gamma^\mu b|\bar{B}(p)\rangle &= f_+(q^2)\left[p^\mu+p'^\mu-\frac{M^2-m_P^2}{q^2}q^\mu\right] \\ &\quad +f_0(q^2)\frac{M^2-m_P^2}{q^2}q^\mu\,, \quad (3.6.1)\\ \langle P(p')|\bar{q}\sigma^{\mu\nu}q_\nu b|\bar{B}(p)\rangle &= \frac{if_T(q^2)}{M+m_P}\left[q^2(p^\mu+p'^\mu)-(M^2-m_P^2)q^\mu\right]\,, \quad (3.6.2)\end{aligned}$$

For vector meson the form factors are given as

$$\begin{aligned}\langle V(p',\varepsilon^*)|\bar{q}\gamma^\mu b|\bar{B}(p)\rangle &= \frac{2iV(q^2)}{M+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p'_\rho p_\sigma\,, \quad (3.6.3)\\ \langle V(p',\varepsilon^*)|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle &= 2m_VA_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu+(M+m_V)A_1(q^2)\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ &\quad -A_2(q^2)\frac{\varepsilon^*\cdot q}{M+m_V}\left[p^\mu+p'^\mu-\frac{M^2-m_V^2}{q^2}q^\mu\right], \quad (3.6.4)\\ \langle V(p',\varepsilon^*)|\bar{q}\sigma^{\mu\nu}q_\nu b|\bar{B}(p)\rangle &= 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho p'_\sigma\,, \quad (3.6.5)\\ \langle V(p',\varepsilon^*)|\bar{q}\sigma^{\mu\nu}\gamma_5 q_\nu b|\bar{B}(p)\rangle &= (-i)T_2(q^2)\left[(M^2-m_V^2)\varepsilon^{*\mu}-(\varepsilon^*\cdot q)(p^\mu+p'^\mu)\right] \\ &\quad +(-i)T_3(q^2)(\varepsilon^*\cdot q)\left[q^\mu-\frac{q^2}{M^2-m_V^2}(p^\mu+p'^\mu)\right], \quad (3.6.6)\end{aligned}$$

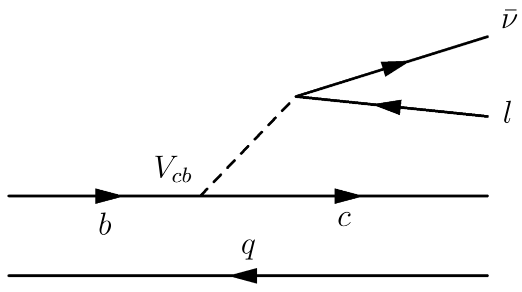


Figure 3.5: Semi-Leptonics decays of B mesons.

where M is the B meson mass, m_p the mass of the pseudoscalar meson and $q = p - p'$ and $m_V(\varepsilon)$ is the mass of the vector meson and the sign convention $\epsilon^{0123} = -1$ has been used.
As long as the velocity transfer to the D meson remains of order 1, one may assume that the heavy quarks interact with the spectator quark (and other soft degrees of freedom) exclusively via soft gluon exchanges characterized by momentum transfers much smaller than the heavy quark masses. Any hard interaction would imply large momentum of the spectator quark in the B meson or D meson or both, and such a configuration is assumed to be highly improbable. However the existence of spin and flavor symmetries in infinite quark mass limit implies that all form factors are related to a single function of velocity transfer, $\xi(v \cdot v')$ known as Isgur-Wise function. The absolute normalization is known at zero recoil ($\xi(v \cdot v') = 1$).

3.6.1 Derivation and discussion of large-recoil symmetries

Heavy-quark symmetry implies relations between the weak decays form factor of heavy meson given in Eq.(3.6.1)-Eq.(3.6.6). In order to calculate them one can write the trace formula give in Eq.(3.5.10) as

$$\langle \bar{\mathcal{M}}'(v') \bar{h}_{v'}^{(Q')} \Gamma h_v^{(Q)} | \mathcal{M}(v) \rangle = -\xi(v \cdot v') \mathrm{tr}\left[\bar{\mathcal{M}}'(v') \Gamma \mathcal{M}(v) \right] \tag{3.6.7}$$

Performing the trace relevant matrix elements for $B \to D(D^*) l \nu$ then can be written as

$$\begin{aligned}
\langle D(v') | \bar{c} \gamma^\mu b | \bar{B}(v) \rangle &= \sqrt{m_B m_D} \left[\xi_+(w)(v^\mu + v'^\mu) \right. \\
&\quad \left. + \xi_-(w)(v^\mu - v'^\mu) \right], & (3.6.8) \\
\langle D(v') | \bar{c} \sigma^{\mu\nu} q_\nu b | \bar{B}(v) \rangle &= -i \sqrt{m_B m_D} \xi_T(w) \left[(v'^\mu (m_D w - m_B) \right. \\
&\quad \left. + v^\mu (m_B w - m_D) \right], & (3.6.9) \\
\langle D^*(v', \epsilon') | \bar{c} \gamma^\mu b | \bar{B}(v) \rangle &= i \sqrt{m_B m_D^*} \xi_V(w) \varepsilon^{\mu\nu\rho\sigma} \epsilon'^*_\nu v'_\rho v_\sigma \,, & (3.6.10) \\
\langle D^*(v', \epsilon^*) | \bar{c} \gamma^\mu \gamma_5 b | \bar{B}(v) \rangle &= \sqrt{m_B m_D^*} \left[\xi_{A1}(w)(w+1) \epsilon'^{*\mu} \right. \\
&\quad \left. - \xi_{A2}(w)(\epsilon^* . v) v'^\mu \right] & (3.6.11) \\
\langle D^*(v', \epsilon^*) | \bar{c} \sigma^{\mu\nu} q_\nu b | \bar{B}(v) \rangle &= \sqrt{m_B m_D^*} \xi_{T1}(w) \varepsilon^{\mu\nu\rho\sigma} \epsilon'^*_\nu v_\rho v'_\sigma (m_B + m_D^*), & (3.6.12) \\
\langle D^*(v', \epsilon'^*) | \bar{c} \sigma^{\mu\nu} \gamma_5 q_\nu b | \bar{B}(v) \rangle &= -i \sqrt{m_B m_D^*} \left[\xi_{T2}(w) \epsilon'^{*\mu} (m_B - m_D^*)(w+1) \right. \\
&\quad \left. - \xi_{T3}(w) m_B \epsilon'^* . v (v^\mu + v'^\mu) \right] & (3.6.13)
\end{aligned}$$

As can be seen in Eq.(3.6.1) that the hadronic matrix element depends upon two general form factors $f_+(q^2)$ and $f_-(q^2)$. In HQET formalism this would corresponds to the existence of two different Lorentz structures $(v+v')^\mu$ and $(v-v')^\mu$. However, since $\bar{h}^{(Q')}_{v'}\,(v-v')\,h^{(Q)}_v = 0$, there is no term proportional to $(v-v')^\mu$. This spin symmetry then also relates the matrix elements of $\bar{B} \to Dl\bar{\nu}$ and $\bar{B} \to D^* l\bar{\nu}$. Thus non-perturbative problem is then reduce to a single form factor $\xi(w)$ which is independent of heavy quark mass. The relations given above can be simplified by taking

$$\begin{aligned} \xi_i(w) &= \xi(w) \quad \text{for } i = +,V,A1,A2,T,T1,T2,T3 \\ \xi_i(w) &= 0 \;\text{ for } i = -,A0 \end{aligned} \tag{3.6.14}$$

The flavor symmetry allows to get the normalization of the Isgur-Wise function. When $v' = v$, the vector current $j^\mu = \bar{h}^{(Q')}_v \gamma^\mu h^{(Q)}_v = \bar{h}^{(Q')}_v v^\mu h^{(Q)}_v$ is conserved:

$$\partial_\mu j^\mu \equiv \int \mathrm{d}^3 x j^0(x) = \int \mathrm{d}^3 v \bar{h}^{(Q')\dagger}_v h^{(Q)}_v \tag{3.6.15}$$

is a generator of the flavor symmetry. Acting over $Q\bar{q}$ meson, it replaces a quark Q by a quark Q': $N_{Q'Q}|P(v)\rangle$. Therefore, it satisfies

$$\langle P'(v)|N_{Q'Q}|P(v)\rangle = \langle P'(v)|P(v)\rangle = 2v^0(2\pi)^3\delta^{(3)}(\vec{0}). \tag{3.6.16}$$

Comparing it with the relation $B \to D^* l\nu$ in Eq.(3.6.8–3.6.13) (taking $\mu = 0$ and integrating over d^3x), one gets the result

$$\xi(w) = 1 \tag{3.6.17}$$

To work out the large-recoil symmetry constraints on the soft form factor one compare Eqs.(3.6.8–3.6.13) with Eqs.(3.6.1–3.6.6). The relations among form factor are then given as :

$$\xi(w) = \frac{2\sqrt{m_B m_D}}{(m_B + m_D)} f_+(q^2) \tag{3.6.18}$$

where

$$f_+(q^2) = \frac{(m_B+m_D)^2}{(2m_B m_D)(v.v'+1)} f_0(q^2) = f_T(q^2) = \xi(\omega) \tag{3.6.19}$$

$$q^2 = m_B^2 + m_D^2 - 2m_B m_D v\cdot v'. \tag{3.6.20}$$

Thus, the heavy-quark flavor symmetry relates two a prior independent form factors to one and the same function. The heavy-quark spin flavor symmetry leads to additional relations among weak decay form factors. It can be used to relate matrix elements involving vector mesons to those involving pseudoscalar mesons. a vector meson with longitudinal polarization is related to a pseudoscalar meson by a rotation of the heavy-quark spin. For vector meson one gets the transformation

$$V(q^2) = \frac{(m_B + m_{D^*})}{2\sqrt{m_B m_D^*}}\xi(\omega) = A_0(q^2) = A_2(q^2) = T_1(q^2) \tag{3.6.21}$$

$$A_1(q^2) = \frac{(\sqrt{m_B m_D^*}(v.v'+1)}{(m_B + m_{D^*})}\xi(\omega) = T_2(q^2)$$

$$T_3(q^2) = \frac{(m_B - m_D^*)}{\sqrt{m_B m_D^*}}\xi(\omega) \tag{3.6.22}$$

the relation among different form factors are given as

$$V(q^2) = \frac{(m_B + m_{D^*})^2}{(2m_B m_D^*)(v.v'+1)}A_1(q^2) = \frac{(m_B + m_{D^*})^2}{(2m_B m_D^*)(v.v'+1)}T_2(q^2)$$
$$= T_1(q^2) = A_2(q^2) = A_0(q^2) = \frac{(m_B + m_D^*)}{2(m_B - m_D^*)}T_3(q^2) = \xi(\omega) \tag{3.6.23}$$

with

$$q^2 = m_B^2 + m_{D^*}^2 - 2m_B m_{D^*} v \cdot v'. \tag{3.6.24}$$

Eq.(3.6.19) and Eq.(3.6.23) summarize the relations imposed by heavy-quark symmetry on the weak decay form factors describing the semi-leptonic decays processes $\bar{B} \to Dl\bar{\nu}$ and $\bar{B} \to D^* l\bar{\nu}$. These relations are model-independent consequences of QCD in the limit where m_b,$m_c \gg \Lambda_{QCD}$. They play a crucial role in the determination of the CKM matrix element $|V_{cb}|$. In terms of the recoil variable $\omega = v \cdot v'$, the differential semi-leptonic decay rates in the heavy-quark limit become [68]

$$\frac{\mathrm{d}\Gamma(\bar{B} \to Dl\bar{\nu})}{\mathrm{d}\omega} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 \left(m_B + m_D\right)^2 m_D^3 \left(\omega^2 - 1\right)^{3/2} \xi^2(\omega),$$
$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^* l\bar{\nu})}{\mathrm{d}\omega} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 \left(m_B + m_{D^*}\right)^2 m_{D^*}^3 \sqrt{\omega^2 - 1}\left(\omega + 1\right)^2 \tag{3.6.25}$$
$$\left[1 + \frac{4\omega}{\omega + 1}\frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{\left(m_B - m_{D^*}\right)^2}\right]\xi^2(\omega)$$

These expressions receive symmetry-breaking corrections, since the masses of the heavy quarks are not infinitely large. Perturbative corrections of order of $\alpha_s^n(m_Q)$ can be calculated order by order in perturbation theory. A more difficult task is to control the non- perturbative power corrections of order $(\Lambda_{QCD}/m_Q)^n$. HQET provides a systematic framework for analyzing these corrections. These perturbative corrections are discussed in next section in detail.
A model independent determination of the CKM matrix element $|V_{cb}|$ based on heavy quark symmetry can be obtained by measuring the recoil spectrum of D^* meson produced in $\bar{B} \to D^* l\nu$ decays. In the heavy quark limit, the differential decay rate for this process has been given in Eq.(3.6.25). In order to allow for the corrections to that limit, one can write

$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^* l\bar{\nu})}{\mathrm{d}\omega} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 \left(m_B - m_{D^*}\right)^2 m_{D^*}^3 \sqrt{\omega^2 - 1}\left(\omega + 1\right)^2$$
$$\left[1 + \frac{4\omega}{\omega + 1}\frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{\left(m_B - m_{D^*}\right)^2}\right]|V_{cb}|^2\mathcal{F}^2(\omega), \tag{3.6.26}$$

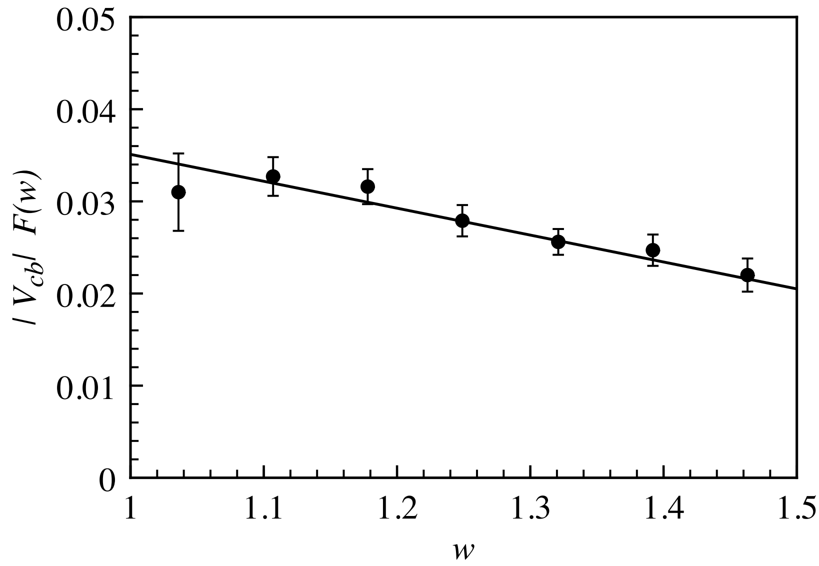


Figure 3.6: CLEO data for the product $|V_{cb}|\mathcal{F}(w)$, as extracted from the recoil spectrum in $\bar{B} \to D^* l\bar{\nu}$ decays. The line shows a linear fit to the data

where hadronic form factor $\mathcal{F}(w)$ coincides with the Isgur-Wise function up to symmetry-breaking corrections of order $\alpha_s(m_Q)$ and Λ_{QCD}/m_Q. The idea was to measure the product $|V_{cb}|\mathcal{F}(\omega)$ as a function of ω, and to extract V_{cb} from the extrapolation of the data to the zero-recoil point $w = 1$, where the B and D^* mesons have common rest frame. At this kinematical point, heavy-quark symmetry helps us to calculate the normalization $\mathcal{F}(1)$ with small controlled errors. Since the range w values accessible in this decay rather small $(1 < w < 1.5)$, the expansion around $w = 1$ is

$$\mathcal{F}(w) = \mathcal{F}(1)\left[1 - \hat{\varrho}^2(w-1) + \hat{c}(w-1)^2 \cdots\right] \tag{3.6.27}$$

The slope $\hat{\varrho}^2$ and the curvature $\hat{c}$, and indeed more generally the complete shape of the form factor, are tightly constrained by analyticity and unitarity requirements. In the long run, the statistics of the experimental results close to zero recoil will be such that these theoretical constraints will not be crucial to get a precision measurement of V_{cb}. They will, however, enable strong consistency checks.

Measurements of the recoil spectrum have been performed by several experimental groups. Fig (3.6) shows, as an example, the data reported sometime ago by the CLEO Collaboration. When the lepton mass is neglected, the differential decay distributions in $\bar{B} \to Dl\bar{\nu}$ decays cab be parametrized by three helicity amplitudes, or equivalently by three independent combinations of form factors. It has been suggested that a good choice for three such quantities should be inspired by heavy quark limit. One thus defines a form factor $h_{A1}(w)$, which up to symmetry- breaking corrections coincides with the Isgur-Wise function, and the two form factor ratios

$$R_1(w) = \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{V(q^2)}{A_1(q^2)}, \tag{3.6.28}$$

$$R_2(w) = \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{A_2(q^2)}{A_1(q^2)}. \tag{3.6.29}$$

As given before $q^2 = m_B^2 + m_{D^*}^2 - 2m_B m_{D^*} v \cdot v'$. This definition is such that in the heavy quark limit $R_1(w) = R_2(w) = 1$ independently of w. To extract the functions $h_{A1}(w)$, $R_1(w)$ and $R_2(w)$ from experimental data is a complicated task. However, HQET-based calculations suggest that w dependence of the form factor ratios, introduced by symmetry breaking effects, is rather mild. Moreover, the form factor $h_{A1}(w)$ is expected to have a nearly linear shape over the accessible w range. This motivates to introduce three parameters $\hat{\varrho}_{A1}^2$, R_1 and R_2 by

$$h_{A1}(w) \sim \mathcal{F}(1) \left[1 - \hat{\varrho}_{A1}^2 (w-1)\right], \tag{3.6.30}$$

$$R_1(w) \sim R_1, \qquad R_2(w) \sim R_2, \tag{3.6.31}$$

where $\mathcal{F}(1)$ is the normalization of $\mathcal{F}(w)$ and can be understood from Eq.(3.6.27) and its value can be given as $\mathcal{F}(1) = 0.91 \pm 0.03$. The CLEO Collaboration has extracted these three parameters from an analysis of the angular distributions in $\bar{B} \to D^* l\nu$ and the results are

$$\hat{\varrho}^2 = 0.91 \pm 0.16, \quad R_1 = 1.18 \pm 0.32, \quad R_2 = 0.71 \pm 0.23. \tag{3.6.32}$$

Using HQET, one obtains an essentially model-independent prediction for the symmetry-breaking corrections to R_1, whereas the corrections to R_2 are somewhat model dependent. To good approximation[18]

$$\begin{aligned} R_1 &\sim 1 + \frac{4\alpha_s(m_c)}{3\pi} + \frac{\bar{\Lambda}}{2m_c} \sim 1.3 \pm 0.1 \\ R_2 &\sim 1 - \kappa \frac{\bar{\Lambda}}{2m_c} \sim 0.8 \pm 0.2 \end{aligned} \tag{3.6.33}$$

with value of $\kappa \sim 1$ from QCD sum rules and $\bar{\Lambda}$ is the binding energy.

Heavy quark symmetry has also been tested by comparing the form factor $\mathcal{F}(w)$ in $\bar{B} \to D^* l\nu$ decays with the corresponding form factor $\mathcal{G}(w)$ governing $\bar{B} \to D l\nu$ decays. The theoretical prediction is

$$\frac{\mathcal{G}(1)}{\mathcal{F}(1)} = 1.08 \pm 0.06 \tag{3.6.34}$$

compares with the experimental results for this ratio 0.99 ± 0.19 reported by CLEO Collaboration and 0.87 ± 0.30 reported by ALEPH Collaboration. In these analysis, it has been tested that within experimental errors the shape of the two form factors agrees over the entire range of w values.

The results of the analysis described above are very encouraging. Within errors, the experiments confirms the HQET predictions, starting to test them at the level of symmetry breaking corrections.

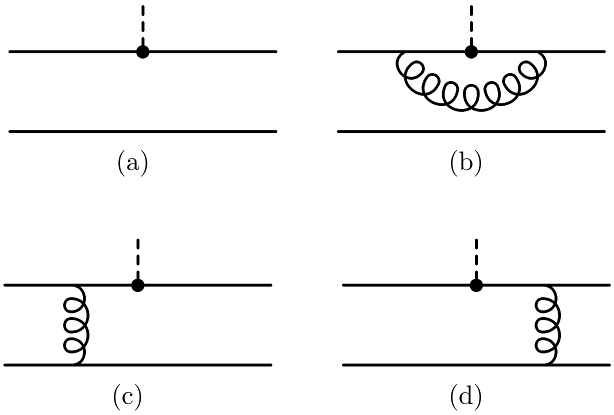


Figure 3.7: Different contributions to the $B \to P(V)$ transitions. (a) Soft contribution. (b) Hard Vertex renormalization.(c,d) Hard spectator interaction.

3.7 Calculations of symmetry-breaking corrections

The general factorization theorem is given in Eq.(3.3.1). However it has been established in [84] that the form factors at large recoil factorize according to

$$f_i(q^2) = C_i \xi(w) + \Phi_B \otimes T_i \otimes \Phi_f \,, \tag{3.7.1}$$

where $\xi(w)$ is the soft part of the form factor which obey the symmetry relations, T_i is the hard-scattering kernel and Φ_B and Φ_f are the light-cone distribution amplitudes for the B and the final state meson f. Furthermore, $\otimes$ denotes a convolution of the hard scattering kernels with the distribution functions.

This factorization formula is used here for a massive collinear charm quark and consider the symmetry breaking corrections in the large recoil region. The aim is to compute the corrections to the symmetry relations (3.6.8)—(3.6.13) that emerge from the second term in (3.7.1) due to the hard scattering.

The distributions functions for the B meson and for the outgoing collinear $D^{(*)}$ meson has been established first. The relevant region under consideration is defined by a small hadronic invariant mass of the final state, $\sqrt{\Lambda_{\rm QCD} m_b} \sim m_c$, while the final state energy is still large of order m_b. This means in particular, that we are close to $w_{\rm max}$ with

$$w_{\rm max} = \frac{1}{2}\left(\frac{m_b}{m_c} + \frac{m_c}{m_b}\right) \gg 1 \tag{3.7.2}$$

The heavy quark symmetries are violated by radiative corrections as shown in Fig(3.7) as well as higher dimension operators in the effective Lagrangian. The disconnected spectator quark line in Fig.(3.7(a)) and (3.7(b)) is meant to indicate that it is connected to the other lines through soft exchanges. Fig.(3.7(a)) is hence the leading term in the heavy quark mass limit.

3.7.1 Vertex Correction

The vertex corrections are shown in Fig.3.7(b) the calculation is identical to the one done in [84]. The one-loop diagram in 3.7(b) contains ultraviolet and infrared divergences. The UV divergences are treated by dimensional regularization ($d = 4-2\epsilon$); the IR divergences are regulated by introducing a small mass term λ for the gluon, and then factored into the soft form factor. Using standard techniques, one obtain for a generic Dirac structure Γ the following result for the one-loop vertex correction is obtained

$$
\begin{aligned}
\bar{u}(p')\Gamma(p,p')u(p) = \frac{\alpha_s C_F}{4\pi}\bar{u}(p')\Bigg[\Bigg\{\frac{1}{2}\ln^2\frac{\lambda^2 m_B^2}{(m_B^2-q^2)^2} - 2\ln\frac{\lambda^2 m_B^2}{(m_B^2-q^2)^2} \\
-2\mathrm{Li}_2\left[\frac{q^2}{m_B^2}\right] - 2\frac{m_B^2}{q^2}\ln\left[1-\frac{q^2}{m_b^2}\right] - 3 - \frac{\pi^2}{2}\Bigg\}\Gamma \\
+\frac{1}{4}\left\{\frac{1}{\hat{\epsilon}} + 3 - \ln\frac{m_B^2}{\mu^2} - \left[1-\frac{q^2}{m_B^2}\right]\ln\left[1-\frac{q^2}{m_B^2}\right]\right\}\gamma^\alpha\gamma^\beta\Gamma\gamma_\beta\gamma_\alpha \\
+\frac{q^2+m_B^2\ln\left[1-\frac{q^2}{m_B^2}\right]}{2q^4}\gamma^\alpha\not{p}\Gamma\not{p}'\gamma_\alpha + \frac{q^2+(m_B^2-q^2)\ln\left[1-\frac{q^2}{m_B^2}\right]}{2q^4}m_B\gamma^\alpha\not{p}\Gamma\gamma_\alpha \\
-\frac{q^2+(m_B^2-2q^2)\ln\left[1-\frac{q^2}{m_B^2}\right]}{q^4}m_B\Gamma\not{p}'\Bigg]u(p),
\end{aligned}
\tag{3.7.3}
$$

where $\bar{u}(p')$ and $\bar{u}(p)$ denote the external Dirac spinors for the c and b quark respectively and $1/\hat{\epsilon} \equiv 1/\epsilon - \gamma_E + \ln 4\pi$. Also $q^2 = m_B^2 + m_D^2 - 2m_B m_D v\cdot v'$ in this case which is different form the value given in [84]. For a given current Γ, the product $\gamma^\alpha\gamma^\beta\Gamma\gamma_\beta\gamma_\alpha$ is evaluated in the naive dimensional regularization scheme with anti commuting γ_5 and the $1/\hat{\epsilon}$ pole is then subtracted details are given in appendix.
The coefficient C_i is Eq.(3.7.1) would normally be obtained by computing the one-loop vertex correction in HQET using same infrared regularization as in the full theory calculations above. The one-loop corrections to C_i is simply the difference between the two calculations and if both theories have the same infrared behavior, C_i must turn out to be independent on the infrared regularization. Here as the effective theory does not correctly reproduce the hard-collinear infrared divergence and all infrared divergences shows the same structure as can be seen from Eq.(3.7.3), so they can simply be absorbed into a redefinition of Isgur-Wise function, irrespective of there origin. In short, the hard-collinear contributions preserve the HQET/large recoil symmetries and can hence be disregarded in the discussion of symmetry-breaking corrections.
So one can then define a factorization scheme by imposing the condition that

$$f_+ \equiv \xi(w), \qquad V \equiv \xi(w) \tag{3.7.4}$$

after subsituting Eq.(3.7.3) into Eq.(3.6.7), we can express all other form factors in terms of $\xi(w)$,

$$f_0 = \frac{2m_B m_D(v.v'+1)}{(m_B+m_D)^2}\xi(w)\left(1+\frac{\alpha_s C_F}{4\pi}[2-2L]\right) + \frac{\alpha_s C_F}{4\pi}\triangle f_0 \tag{3.7.5}$$

$$f_T = \xi(w)\left(1+\frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_B^2}{\mu^2}+2L\right]\right) + \frac{\alpha_s C_F}{4\pi}\triangle f_T \tag{3.7.6}$$

for the remaining form factors of pseudoscalar mesons

$$A_1 = \frac{2m_B m_D^*(v.v'+1)}{(m_B+m_D^*)^2}\xi(w) + \frac{\alpha_s C_F}{4\pi}\triangle A_1, \tag{3.7.7}$$

$$A_2 = \xi(w) + \frac{\alpha_s C_F}{4\pi}\triangle A_2, \tag{3.7.8}$$

$$T_1 = \xi(w)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_B^2}{\mu^2} - L\right]\right) + \frac{\alpha_s C_F}{4\pi}\triangle T_1, \tag{3.7.9}$$

$$T_2 = \frac{2m_B m_D^*(v.v'+1)}{(m_B+m_D^*)^2}\xi(w)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_B^2}{\mu^2} - L\right]\right) + \frac{\alpha_s C_F}{4\pi}\triangle T_2, \tag{3.7.10}$$

$$T_3 = \frac{(m_B - m_{D^*})}{m_B + m_{D^*}}\xi(w)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_B^2}{\mu^2} - L\right]\right) + \frac{\alpha_s C_F}{4\pi}\triangle T_3 \tag{3.7.11}$$

for remaining form factors of vector meson. Where

$$L = -\frac{2m_B m_D(v\cdot v'+1)}{m_B^2 + m_D^2 - 2m_B m_D v\cdot v'}\ln\frac{2m_B m_D(v\cdot v'+1)}{(m_B+m_D)^2} \tag{3.7.12}$$

has been used. The form factors will now receives a further additive correction from the interaction with the spectator quark, indicated by $\triangle F_i$ in Eq.(3.7.5–3.7.6) and Eq.(3.7.7–3.7.11). This correction is calculated in the next subsection.

3.7.2 Hard spectator interaction

As stated earlier, further corrections at α_s order arises from hard spectator interactions given in Fig 3.7(c) and Fig 3.7(d). These calculations can be done by the use of two-particle light-cone distribution amplitudes of B and D mesons. The momentum of the b quark is given as

$$p_b^\mu = m_B v^\mu, \qquad l^\mu = \frac{l_+}{2}n_+^\mu + l_\perp^\mu + \frac{l_-}{2}n_-^\mu \tag{3.7.13}$$

where p_B^μ is the momentum of the b quark and l^μ is the momentum of the light spectator quark. Note that all components of l^μ are of order $\Lambda_{\rm QCD}$ in rest frame of B-meson. One can write a similar decomposition valid in the rest frame of the $D(D^*)$ meson in terms of two new light-cone vectors

$$p_c'^\mu = m_D v'^\mu, \qquad k'^\mu = \frac{k'_+}{2}n_+'^\mu + k_\perp'^\mu + \frac{k'_-}{2}n_-'^\mu \tag{3.7.14}$$

where now $p_D'^\mu$ is the momentum of the c quark and k'^μ is the momentum of the spectator quark in the $D^{(*)}$-meson. It is to kept in mind that the k'^μ has the momentum of order $\Lambda_{\rm QCD}$ in D-meson rest frame. In order to avoid confusion primed coordinates has been used for D-meson rest frame and the unprimed for B-meson rest frame with the corresponding velocities vectors satisfy $v' = \frac{1}{2}(n'_+ + n'_-)$ and $v' = \frac{1}{2}(n'_+ + n'_-)$. With two different rest frames having velocities v and v', two sets of light-cone vectors can be defined as

$$n_\pm = v \mp \frac{v' - (v.v')v}{\sqrt{(v.v')^2 - 1}} \tag{3.7.15}$$

for the one's in the rest frame of B-meson and

$$n'_{\pm} = v' \mp \frac{v - (v.v')v'}{\sqrt{(v.v')^2 - 1}} \tag{3.7.16}$$

in the rest frame of D-meson. With this it has to be kept in mind that k'^{μ} in the rest frame of B- meson will be decomposed as

$$\frac{k_+}{2} n_+^{\mu} + k_{\perp}^{\mu} + \frac{k_-}{2} n_-^{\mu} \tag{3.7.17}$$

with $k_+ = n'_-.\vec{k'}$. As the result the $k_+ \approx \frac{1}{2(v.v')} k'_-$ and likewise $k_- \approx 2(v.v')k'_+$. Hence the momentum power counting for k'^{μ} in B-meson rest frame will be ($\Lambda_{\rm QCD}^{3/2}$, $\Lambda_{\rm QCD}$ $\Lambda_{\rm QCD}^{1/2}$) i.e., in the B-meson rest frame the k_- component is leading. Similar power change is obtained for l^{μ} in D-meson rest frame.
The contribution of the hard spectator interaction given in Fig.(3.7(c)) and (3.7(d)) to the current matrix elements is give by the convolution formula

$$\langle L|\bar{q}\Gamma b|B\rangle = \frac{4\pi\alpha_s C_F}{N_C} \int_0^{\infty} \mathrm{d}k'_+ \int_0^{\infty} \mathrm{d}l_+ M^B_{\beta\gamma} M^D_{\delta\alpha} \mathcal{T}^{\Gamma}_{\alpha\beta\gamma\delta} \tag{3.7.18}$$

where Γ denotes an arbitrary Dirac matrix, and $\tau^{\Gamma}_{\alpha\beta\gamma\delta}$ is hard-scattering amplitude which will be calculated from the Feynman graphs in Figs. 3.7(c),3.7(d). The color traces has already been performed in the above written convolution formula. The initial and final state mesons are written in term of the two-particle light-cone projectors $M^B_{\beta\gamma}$,$M^D_{\delta\alpha}$. The expressions for light-cone projectors for heavy meson are obtained after Fourier transformation to momentum space of the light-cone expansion of matrix elements of the quark-anti quark operators (discussed in detail in appendix). The projector needed can then be written as

$$M^B_{\beta\gamma} = -\frac{if_B m_b}{4}\left[\frac{1+\not{v}}{2}\left\{\phi^B_+(l_+)\not{n}_+ + \phi^B_-(l_+)\left(\not{n}_- - l_+\gamma^{\nu}_{\perp}\frac{\partial}{\partial l^{\nu}_{\perp}}\right)\right\}\gamma_5\right]_{\beta\gamma}\Big|_{l=\frac{l_+}{2}n_+} \tag{3.7.19}$$

Similarly for the final state D-meson the light-cone projector is

$$M^D_{\beta\gamma} = -\frac{if_D M_D}{4}\left[\frac{1+\not{v}'}{2}\left\{\phi^D_+(k'_+)\not{n}'_+ + \phi^B_-(k'_+)\left(\not{n}'_- - k'_+\gamma^{\nu}_{\perp}\frac{\partial}{\partial k'^{\nu}_{\perp}}\right)\right\}\gamma_5\right]_{\beta\gamma}\Big|_{k'=\frac{k'_+}{2}n_+} \tag{3.7.20}$$

In Feynman gauge, the hard-scattering amplitude is given by the expression

$$\mathcal{T}^{\Gamma}_{\alpha\beta\gamma\delta} = \left[\Gamma\left(\frac{(m_B\not{v} + \not{l} - \not{k}) + m_B}{(m_B v + l - k)^2 - m_B^2}\right)\gamma_{\mu} + \gamma_{\mu}\left(\frac{(m_D\not{v}' - \not{l} + \not{k}') + m_D}{(m_D v' - l + k)^2 - m_D^2}\right)\Gamma\right]_{\alpha\beta}\frac{1}{(l-k)^2}[\gamma^{\mu}]_{\gamma\delta} \tag{3.7.21}$$

The hard scattering contributions to the current matrix elements for $\bar{B} \to Dl\bar{\nu}$ transitions can be calculated by using Eq.(3.7.18) and Eq.(3.7.21) together with the light-cone projection operators. The form factors are then determined by comparing the result with the definitions in Eq.(3.6.1)– Eq.(3.6.6). After calculating trace and comparison, the result for the form factor $f_+(q^2)$ reads

$$f_+^{HSA} = \frac{\alpha_s C_F}{4N_c}\frac{(m_b+m_c)f_B f_D}{(v\cdot v')^2}\int\limits_0^\infty \mathrm{d}l_+ \int\limits_0^\infty \mathrm{d}k'_+ \frac{\phi_+^D \phi_-^B}{l_+ k'^2_+} \tag{3.7.22}$$

The corresponding expression for f_0 is

$$f_0^{HSA} = \frac{\alpha_s C_F}{4N_c}\frac{(m_b m_c)}{(m_b+m_c)}\frac{f_B f_D(v\cdot v'+1)}{(v\cdot v')^2}\int\limits_0^\infty \mathrm{d}l_+ \int\limits_0^\infty \mathrm{d}k'_+ \frac{\phi_+^D \phi_-^B}{2l_+ k'^2_+} \tag{3.7.23}$$

As can be seen that these terms have endpoint singularities for $l_+ \to 0$ and/r $k'_+ \to 0$. But there is no contribution of the form

$$I = \text{Constant}\int\limits_0^\infty \frac{\mathrm{d}l_+ \phi_-^B}{l_+}\int\limits_0^\infty \frac{\mathrm{d}k'_+ \phi_+^D}{k'_+} \tag{3.7.24}$$

Contributions like this are computable with hard scattering methods and hence gives rise to symmetry breaking corrections, as in order of α_s one only needs particular moments of the distribution amplitudes to compute corrections. Same as done in [84] for the case of $B \to \pi l\nu$. There the hard spectator interaction contributions are given by using the moment for B and light pseudoscalar meson

But in $\bar{B} \to D l\bar{\nu}$ case there is no term having structure given in Eq.(3.7.24). The terms given in Eq.(3.7.22) preserve the symmetry structure given in Eq.(3.6.7). In factorization scheme discussed here these terms can be absorbed in $\xi(\omega)$ by using the renormalization convention adopted in Eq.(3.7.4) (as the terms with this structure cannot be computed with the standard hard scattering methods. Hence the result for the hard scattering correction for $\bar{B} \to D l\bar{\nu}$ form factors as defined by $\triangle f_{+,0,T}$ in Eq.(3.7.5) can be calculated. The renormalization convention Eq.(3.7.4) implies that $\triangle f_+ \equiv 0$ by definition. It is interesting to note that in case of $\bar{B} \to D l\bar{\nu}$ $\triangle f_0 \equiv 0$ and $\triangle f_T \equiv 0$. This is unlike to what is predicted.
The analysis of the hard-scattering corrections to form factors for $\bar{B} \to D^* l\bar{\nu}$ transitions proceeds in the same way as for $\bar{B} \to D l\bar{\nu}$. Here as well the contributions obtained are absorbed by renormalization convention Eq.(3.7.4). So there will be no symmetry breaking contributions from hard-scattering interactions. Hence at order α_s symmetry breaking corrections are only coming from the vertex corrections and there is no additional contributions coming from the hard spectator interactions as was predicted to present.

The symmetry breaking corrections to semi-leptonic form factors, calculated above, can be tested experimentally by comparing the different form factor describing the decays at the same value of w.

Chapter 4

Conclusion and outlook

Left right symmetric models are studied by adding an additional family symmetry which is chosen to $U(1)_{family}$. With the few conditions applied on the charges $U(1)_{family}$, CKM matrix is obtained which is comparable with the models already discussed in literature specially by Nir et al. One of the very important consequence of the study of LRSMs is the availability of right handed currents which are absent in Standard Model making it possible to study chirality of the $b \to c$ decays in heavy quark effective theories.

Second portion of the work comprises of the study of symmetry breaking corrections to $\bar{B} \to D(D^*)l\bar{\nu}$ with the help effective field theory methods at large recoil regime. SCET has been considered to include a massive collinear charm quark by defining a power counting $\lambda \sim m_c/m_b \sim \sqrt{\Lambda_{QCD}/m_b}$. Taking into account this power counting a factorization formula for heavy- to -heavy form factors, to separate long and short distance physics, is tested using a toy integral for endpoint singularities. The toy integral is expanded in momentum regions and three different regions hard-collinear, soft (HQET) and soft regions are obtained having non-vanishing contributions. All three regions got end point divergences however soft (HQET) and soft regions requires an additional regularization as they are soft and are not well defined in dimensional regularization. As the result soft(HQET) and soft regions gives rise to an additional singularity in δ along with the usual singularity in ϵ, however, all these singularities get canceled when added thus giving a finite result. As factorization formula hold so it means the effects of heavy particles and/or highly virtual radiative corrections can be calculated in perturbative QCD, while the long-distance physics of light quarks and gluons can be encoded in hadronic matrix elements. After testing the factorization formula long distance physics is studied for exclusive $\bar{B} \to D(D^*)l\bar{\nu}$ decays at large recoil presented by hadronic form factors.

Heavy spin symmetry for heavy-to-heavy decays relates pseudoscalar and vector mesons as all independent form factors for pseudoscalar as well as vector mesons transitions reduces to a single Isgur-Wise function $\xi(w)$ which is independent on the heavy quark mass. Before studying symmetry breaking corrections, symmetry relations among form factors for $\bar{B} \to D(D^*)l\bar{\nu}$ has been calculated at large recoil energy.
Symmetry breaking effects come from hard gluon corrections and falls into two

classes: vertex corrections to the heavy-to-heavy current and hard re-scattering with the spectator quark which is described by the hard-scattering approach and involves light-cone distribution amplitudes of the participating mesons. The vertex corrections has been treated in analogous way as heavy quark effective theory and has the results similar to the heavy-to-light current with the redefinition of momentum transfer parameter. For hard scattering interactions with spectator quark it was expected that the structure of heavy quark/large recoil symmetries survives radiative corrections (in the sense that symmetry breaking effects should be dominated by hard scattering and therefore computable with the standard methods) as in the heavy-to-light case. However it is observed that for the case of heavy-to-heavy form factors, there are no computable corrections from the hard spectator interactions. As a result the complete symmetry breaking corrections to leading order in α_s are coming only form vertex interactions.

On the basis of this work a detailed test of right-hand currents may be performed. While most analysis concentrate on the non recoil point $v = v'$, the present calculation opens the door to include the full phase space of $b \to c$ transitions. The detailed analysis involves a complete refitting of the experimental information and is beyond the scope of this theoretical work. In order to perform a precision analysis one could extend the present work by using more sophisticated heavy-quark-expansion or soft-collinear-effective-theory methods, including also perturbative and non-perturbative corrections.

Appendix A

A.1 Calclution of vertex correction

Calculations for the vertex corrections given in Eq.(3.7.3)[84] are done here in somewhat detail. The one loop tensor integrals, in D dimensions are classified according to the number N of propagator factors in the denominator and the number P of integration momenta in the numerator. In general they are defined as

$$T^N_{\mu_1\cdots\mu_P}(p1,\cdots,p_{N-1},m_0,\cdots m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q \frac{q_{\mu_1}\cdots q_{\mu_P}}{D_0 D_1\cdots D_{N-1}} \quad \text{(A.1.1)}$$

with denominator factors as

$$D_0 = q^2 - m_0^2 + i\epsilon, \quad D_i = (q+p_i)^2 - m_i^2 + i\epsilon, \quad i = 1,\cdots,N-1 \quad \text{(A.1.2)}$$

T^N is denoted Nth character of the alphabet, i.e. $T^1 \equiv A$,$T^2 \equiv B,\cdots,$. Lorentz covariance of the integral allows one to decompose the tensor integrals into tensors constructed from the external momenta p_i, and a metric tensor $g_{\mu\nu}$ with totally symmetric coefficient functions $T^N_{i_1,\cdots,i_P}$. These integrals are UV divergent and these divergences are regularized by calculating the integrals in D dimension with $D = 4-2\epsilon$. The parameter μ has mass dimension and serves to keep the dimension of the integrals fixed for varying D(details can be seen in [93]).
With momenta assigned as shown in Fig.(A.1) application of Feynman rules for some general current Γ one finds

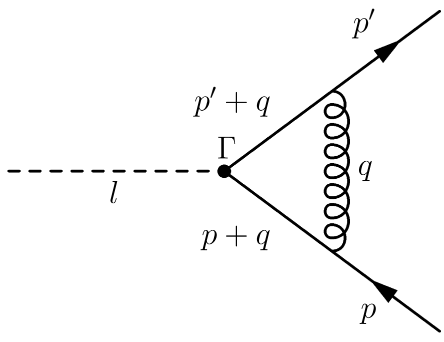


Figure A.1: QCD Vertex correction to $b \to u(l\nu)$ decays

$$\bar{u}(p')\Gamma(p,p')u(p) = -ig_sC_F\mu^{2\epsilon}\int\frac{\mathrm{d}^Dq}{(2\pi)^D}$$

$$\frac{\bar{u}(p')\gamma_\alpha\left(\not{q}+\not{p}'+m_u\right)\Gamma\left(\not{q}+\not{p}+m_b\right)\gamma^\alpha}{((p'+q)^2-m_u^2+i\epsilon)\left((p+q)^2-m_b^2+i\epsilon\right)(q^2-\lambda^2+i\epsilon)\,u(p)} \tag{A.1.3}$$

This expression can be simplified using Dirac identities and equations of motion $(\not{p}-m_b)u(p) = (\not{p}'-m_u)u(p') = 0$ and setting $m_u \sim 0$,

$$\bar{u}(p')\Gamma(p,p')u(p) = -ig_sC_F\mu^{2\epsilon}\int\frac{\mathrm{d}^Dq}{(2\pi)^D}$$

$$\frac{\bar{u}(p')\left[\gamma_\alpha\not{p}'\Gamma\not{p}\gamma^\alpha+\gamma_\alpha\not{p}'\Gamma\gamma^\alpha m_b+\gamma_\alpha\not{q}\Gamma\not{p}\gamma^\alpha+\gamma_\alpha\not{p}'\Gamma\not{q}\gamma^\alpha+\gamma_\alpha\not{q}\Gamma\gamma^\alpha m_b+\gamma_\alpha\not{q}\Gamma\not{q}\gamma^\alpha\right]u(p)}{((p'+q)^2+i\epsilon)\left((p+q)^2-m_b^2+i\epsilon\right)(q^2-\lambda^2+i\epsilon)} \tag{A.1.4}$$

First two terms in the numerator are scalar as they do not contain loop momentum q so they can be contracted by the use of anti commutation relations and the Dirac equation, while the other terms containing loop momenta can be simplified by the use of method(give name) After using the above mentioned process in Eq.(A.1.4) the numerator read as

$$\begin{aligned}&= \bar{u}(p')\left[\left(4D^2p'\cdot pC_0+2C_1m_b^2+(C_1+C_2)p'\cdot p+\right.\right.\\ &\qquad\left.C_{11}m_b^2+C_{12}p'\cdot p\right)\Gamma-\left(C_1+C_{12}\right)m_b\Gamma\not{p}'\\ &\qquad\left.-C_{11}m_b\gamma_\alpha\not{p}\Gamma\gamma^\alpha+C_{12}\gamma_\alpha\not{p}\Gamma\not{p}'\gamma^\alpha+C_{00}\gamma_\alpha\gamma^\beta\Gamma\gamma_\beta\gamma^\alpha C_{00}\right]u(p)\end{aligned}$$

Now C_1,C_2,C_{11},$\cdots$ are tensor integral which can be reduced by using the method of $\cdots$ to scalar integrals C_0,$B_0\cdots$. These scalar integrals are then easy to calculate by the use of Feynman parameters. As an example C_0 is calculated herein detail all other can be done in same manner

$$C_0 = -ig_sC_F\mu^{2\epsilon}\int\frac{\mathrm{d}^Dq}{(2\pi)^D}\left\{\left(q^2-\lambda^2+i\epsilon\right)\left((q+p')^2-m_u^2+i\epsilon\right)\left((q+p)^2-m_b^2+i\epsilon\right)\right\}^{-1} \tag{A.1.5}$$

Use of Feynman parameters i.e.,

$$\frac{1}{abc} = \int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{2\theta(1-x-y)}{\left[a(1-x-y)+bx+cy\right]^3} \tag{A.1.6}$$

with $a = \left(q^2-\lambda^2+i\epsilon\right)$,$b = \left((q+p')^2-m_u^2+i\epsilon\right)$ and $c = \left((q+p)^2-m_b^2+i\epsilon\right)$. After simplifying it and shifting $q \to q+(1-x)p$ to complete the square one gets an integral of the form

$$C_0 = -ig_sC_F\mu^{2\epsilon}\int\frac{\mathrm{d}^Dq}{(2\pi)^D}\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{2\theta(1-x-y)}{\left[q^2-M_x^2\right]^3} \tag{A.1.7}$$

with $M_x = x\lambda^2+(1-x)^2m_b^2$. Integration over the loop momentum can be performed by using the master inetgral i.e,

$$\int\frac{\mathrm{d}^Dk}{(-k^2+M_x^2-i0)^n} = i\pi^{D/2}(M_x^2)^{D/2-n}\frac{\Gamma(n-D/2)}{\Gamma(n)} \tag{A.1.8}$$

Here the value of $n = 3$. Using this relation for master integral and setting $D = 4-2\epsilon$ one gets

$$C_0 = -g_s C_F \mu^{2\epsilon} \pi^{2-\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(3)} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y (x\lambda^2 + m_b^2(1-x)^2)^{-(1+\epsilon)} \tag{A.1.9}$$

It is then quite straight forward to do the integration for y and x. And then after expansion in ϵ one gets the result for C_0.

A.2 Derivation of quark-anti quark wavefunctions

A.2.1 The momentum space projector

B- meson projection operator given in Eq.(3.7.19) is derived here. Starting from two-particle [84] light-cone matrix element in coordinate space, two functions $\tilde{\phi}^B_{\pm}(t)$ has been introduced through the Lorentz decomposition of the light-cone matrix element given as

$$\langle 0|\bar{q}_\beta(z)P(z,0)b_\alpha(0)|\bar{B}(p)\rangle = -\frac{if_B M}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\phi}^B_+(t) + \frac{\tilde{\phi}^B_-(t)-\tilde{\phi}^B_+(t)}{t}\not{z}\right\}\gamma_5\right]_{\alpha\beta} \tag{A.2.1}$$

where z is a null vector on light cone ($z^2=0$), $t = v\cdot z$ and $p = Mv$ is the momentum of B-meson with mass M. $b_\alpha(0)$ denotes the effective b-quark field and

$$P(z_2,z_1) = Pexp\left(ig_s\int_{z_2}^{z_1} \mathrm{d}z^\mu A_\mu(z)\right) \tag{A.2.2}$$

is string operator ensuring the gauge invariance. In the light-cone gauge ($A_+ = 0$), one simply has $P(z,0) = 1$. The prefactor is chosen as

$$\langle 0|\bar{q}_\beta[\gamma^\mu\gamma_5]b_\alpha(0)|\bar{B}(p)\rangle = if_B M v^\mu \tag{A.2.3}$$

for $z=0$ so that $\tilde{\phi}^B_{\pm}(t=0) = 1$. $\tilde{\phi}^B_+$ is of leading twist , where as $\tilde{\phi}^B_-$ has sub leading twist[94] [95].
By taking $M(z)$ the matrix element in Eq.(A.2.1) and $A(z)$ the hard scattering amplitude in coordinate space (represented as $A(l)$ in momentum space), One can obtain the momentum space projector M^B of Eq.(3.7.19) through the identity

$$\int \mathrm{d}^4z M(z)A(z) = \int \frac{\mathrm{d}^4 l}{(2\pi)^4}A(l)\int \mathrm{d}^4z e^{-ilz}M(z) \tag{A.2.4}$$

with the decomposition of l^μ to be taken as $\frac{l_+}{2}n^\mu_+ + \frac{l_-}{2}n^\mu_- + l^\mu_\perp$ this gives

$$\int \mathrm{d}^4z M(z)A(z) \equiv \int_0^\infty \mathrm{d}l_+ M^B A(l)|_{l=\frac{l_+}{2}n_+} \tag{A.2.5}$$

The factors $\not{z}$ and $1/(v\cdot z)$ appearing in Eq.(A.2.1) can be removed by having a derivative act on the hard scattering amplitude, and by partial integration. As can be seen the wave functions $\tilde{\phi}^B_{\pm}(t)$ depends upon the separation t on light cone. The corresponding wave functions in momentum space can be defined as

$$\phi_\pm(\omega) = \frac{1}{2\pi}\int \mathrm{d}t\tilde{\phi}_\pm(t)e^{i\omega t}, \quad \tilde{\phi}_\pm(t) = \frac{1}{2\pi}\int \mathrm{d}t\phi_\pm(t)e^{-i\omega t} \tag{A.2.6}$$

The variable ω has the meaning of the light-cone projection p_+ of the light-quark momentum in the heavy meson rest frame. The positions of singularities in the complex t plane are such that $\phi_\pm(\omega)$ vanish for $\omega > 0$. The wave functions are normalized such that

$$\tilde{\phi}(0) = \int_0^\infty \mathrm{d}\omega \phi_\pm(\omega) = 1. \tag{A.2.7}$$

So one can then write

$$\int \mathrm{d}^4 z M(z) A(z) = -\frac{if_B M}{4}\left[\frac{1+\not{v}}{2}\int_0^\infty \mathrm{d}\omega \left\{2\phi_+^B(\omega) - \int_0^\omega \mathrm{d}\eta (\phi_-^B(\eta) - \phi_+^B(\eta))\gamma^\mu \frac{\partial}{\partial l_\mu}\right\}\gamma_5\right]_{\beta\alpha} A(l)_{\alpha\beta}\,|_{l=\omega v}\,.$$

This expression is very closed to the expression desired except that one need to set $l = \omega v = \omega(n_+ + n_-)/2$. The hard scattering amplitude $A(l)$ for a decay into an energetic light meson moving in the n_- direction has the property that it is independent of l_- at leading order in the heavy quark expansion. It can be written more precisely as

$$A(l) = A^{(0)}(l_+) + l_\perp^\mu A^{(1)}(l_+) + \mathcal{O}(1/M). \tag{A.2.8}$$

Hence the n_- component of v does not contribute and one can set $l = \omega n_+/2$. Using

$$\frac{\partial}{\partial l_\mu} = n_-^\mu \frac{\partial}{\partial l_+} + n_+^\mu \frac{\partial}{\partial l_-} + \frac{\partial}{\partial l_{\perp\mu}}, \tag{A.2.9}$$

one obtain

$$M_{\beta\alpha}^B = -\frac{if_B M}{4}\left[\frac{1+\not{v}}{2}\left\{\phi_+^B(\omega)\not{n}_+ + \phi_-^B(\omega)\not{n}_- - \int_0^{l_+} \mathrm{d}\eta \left(\phi_-^B(\eta) - \phi_+^B(\eta)\right)\gamma^\mu \frac{\partial}{\partial l_{\perp\mu}}\right\}\gamma_5\right]_{\alpha\beta} \tag{A.2.10}$$

where the derivative with respect to l_- has been dropped. It is to make clear that $l = l_+ n_+/2$ is set after taking derivative.

A.2.2 Equation of motion constraint

The B meson light-cone projector assumes the form quoted in Eq.(3.7.19) after implementing the equation of motion constraint which are derived in this sub section. In order the derive these relations the equation of motion of light quark in Eq.(A.2.1) is employed. Since the derivative with respect to z_μ has to be taken before the limit $z^2 \to 0$, the definitions in Eq.(A.2.1) is extended to the case $z^2 \neq 0$ via $\tilde{\phi}_\pm^B(t) \to \tilde{\phi}_\pm^B(t,z^2)$. Requiring the right-hand side in Eq.(A.2.1) to vanish after application of $[\not{\partial}_{z_2}]_{\beta\gamma}$ and requiring $\tilde{\phi}_\pm^B(t,z^2)$ to not vanish as $z^2 \to 0$ one obtain

$$\frac{\partial \tilde{\phi}_-^B}{\partial t} + \frac{1}{t}\left(\tilde{\phi}_-^B - \tilde{\phi}_+^B\right)|_{z^2=0} = 0, \tag{A.2.11}$$

$$\frac{\partial \tilde{\phi}_-^B}{\partial z^2} + \frac{1}{4}\frac{\partial^2 \tilde{\phi}_-^B}{\partial t^2}|_{z^2=0} = 0. \tag{A.2.12}$$

The first equation gives the desired relations between $\tilde{\phi}_+^B$ and $\tilde{\phi}_-^B$ in coordinate space.In terms of the momentum space distribution amplitudes, in Eq.(A.2.11) reads

$$\int_0^{l_+} d\eta \left(\phi_-^B(\eta) - \phi_+^B(\eta)\right) = l_+ \phi_-^B(l_+) \qquad \text{or} \qquad \phi_+^B(l_+) = -l_+ \phi_-'^B(l_+), \tag{A.2.13}$$

which is solved by

$$\phi_-^B(l_+) = \int_0^1 \frac{d\eta}{\eta} \phi_+^B(l_+/\eta). \tag{A.2.14}$$

In terms of Mellin moments one has for $N \geq 1$

$$\langle l_+^{N-1} \rangle_+ = N \langle l_+^{N-1} \rangle_-, \qquad \left[\langle l_+^{N-1} \rangle_\pm \equiv \int_0^\infty dl_+^{N-1} \phi_\pm^B(l_+) \right] \tag{A.2.15}$$

The relation (A.2.15) has been derived for $N = 2$ and $N = 3$ in complete detail in [94] from the equations of motions for heavy quark and Lorentz invariance. This gives

$$\langle l_+ \rangle_+ = \frac{4}{3}\bar{\Lambda}, \qquad \langle l_+ \rangle_- = \frac{2}{3}\bar{\Lambda}. \tag{A.2.16}$$

where $\bar{\Lambda} = M - m_b$ is the leading contribution to mass difference in the HQET. For $N = 3$ similar analysis gives [94]

$$\langle l_+^2 \rangle_+ = 2\bar{\Lambda}^2 + \frac{2}{3}\lambda_E^2 + \frac{1}{3}\lambda_H^2, \qquad \langle l_+^2 \rangle_- = \frac{2}{3}\bar{\Lambda}^2 + \frac{1}{3}\lambda_H^2. \tag{A.2.17}$$

where λ_H and λ_E parametrize the contributions of the chromomagnetic and chromoelectric fields to the mass difference $M - m_b$.

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